Hydrodynamics of 2-group global symmetry

Nick Poovuttikul

Department of mathematical sciences,

Based on work 2010.00320 with Nabil Iqbal

Conservation laws with extra assumptions

(Unreasonably) amazing EFT of
IR physics at finite temperature

It behave somewhat like a group
but is NOT a group

Generalised structure found in
studies of higher-category

Hydrodynamics of 2-group global symmetry

Baez & Huerta '10

Cordova's talk at strings 2020

Genuine symmetry of the system
(not redundancy)

Can be broken, emergent, gauged, anomalous

If continuous, implies existence of conserved currents

What is hydrodynamics

* It is a gradient expansions of **Noether currents**

$$Z[g_{\mu\nu}, a_\mu] = \left\langle \exp\left[i \int d^{d+1}x \sqrt{-g} \left(\frac{1}{2} T^{\mu\nu} g_{\mu\nu} + j^\mu a_\mu \right) \right] \right\rangle$$

* Expressed them in terms of macroscopic variables (conjugated to conserved currents)

Fluid velocity

$$j^\mu = n(T, \mu) u^\mu + \mathcal{O}(\partial)$$

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu} + \mathcal{O}(\partial)$$

Local thermodynamic variables

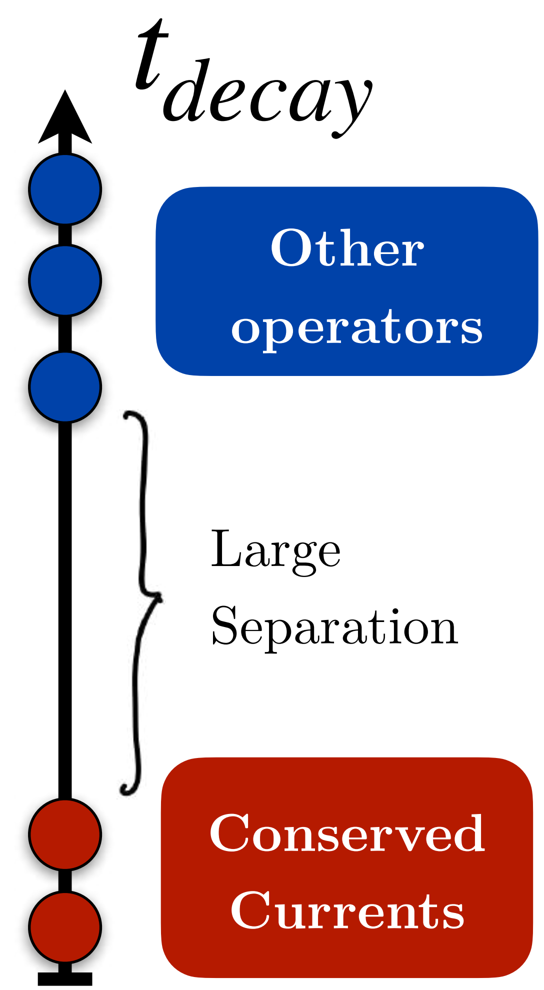
c.f. Landau & Lifshitz book

What is hydrodynamic limit?

* All operators decays much faster
  than scale of interest except conserved currents

* No branch cut near at small ω, k or at late time
  All 1-pt functions at late time $\sim exp(-t/t_{decay})$

* Theories with same global symmetries
  can be describe by the same equations!



Hydrodynamics for/from Strongly coupled QFTs

Using hydrodynamics as a starting point for strong interaction
Here are some dreams

- ✻ Given **any** (global)symmetry of the system in the IR, can we one always derive hydrodynamics as an macroscopically consistent EFT that works better and better at strong interaction?

- ✻ Using non-perturbative technique to understand the "deviation" from classical hydrodynamics. Why it works? When should we trust it?

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How to build (weirder) symmetry structure?

A theory with two conserved U(1)

* Consider the partition function in (3+1)-d spacetime

$$Z[a_\mu, v_\mu] = \left\langle \exp\left[i \int d^{3+1}x \sqrt{-g}\, \left(j_a^\mu a_\mu + j_v^\mu v_\mu \right) \right] \right\rangle$$

If Z invariant under

$$a \to a + d\lambda\,, \qquad v \to v + d\tilde{\lambda}$$

Then we have

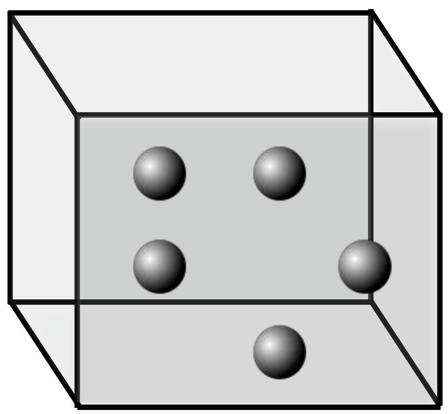
$$\partial_\mu \langle j_a^\mu \rangle = 0\,, \qquad \partial_\mu \langle j_v^\mu \rangle = 0$$

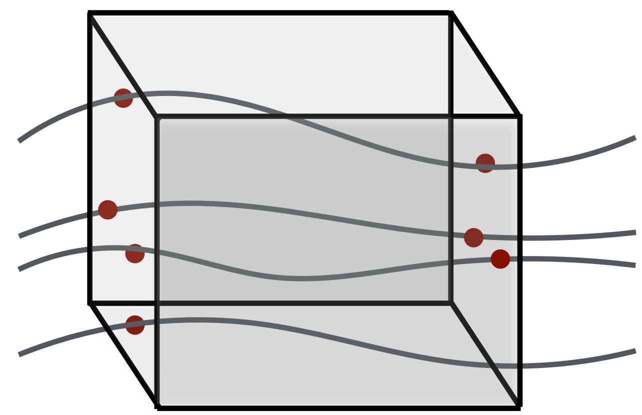
* If Z is invariant up to phase, we have anomaly

  - Conservation law is broken, but in a manageable way

Global symmetry after gauging

* Without anomaly, we say that "particle" (of type a) and magnetic flux are conserved





$$\partial \langle j_a^\mu \rangle = 0$$

Not conserved

~~$\partial_\mu \langle F^{\mu\nu} \rangle = j^\nu$~~

$$\partial_\mu \langle J^{\mu\nu} \rangle = 0$$

* Partition function invariant under

$$a \to a + d\lambda$$
$$b \to b + d\Lambda$$

$$Z[a_\mu, b_{\mu\nu}] = \left\langle \exp\left[i \int d^{3+1}x \sqrt{-g} \left(j_a^\mu a_\mu + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

Anomaly and gauging

- There are four types of anomalies via $Z_{inv} = Z_{anom} e^{iS_{top}}$

$$S_{top}[da, dv] \sim \int d^6X \Big(\kappa_{v^3}\, dv \wedge dv \wedge dv + \kappa_{v^2 a}\, dv \wedge dv \wedge da + \kappa_{va^2}\, dv \wedge da \wedge da + \kappa_{a^3}\, da \wedge da \wedge da \Big)$$

The modified conservation law is

$$d \star \langle j_a \rangle = \kappa_{a^3} da \wedge da + \kappa_{a^2 v} da \wedge dv + \kappa_{av^2} dv \wedge dv$$

$$d \star \langle j_v \rangle = \kappa_{v^3} dv \wedge dv$$

- They are obstruction of gauging i.e. imposing $\partial_\mu \langle F \rangle^{\mu\nu} = \langle j^\nu \rangle$
  - a^3 type : cannot gauge $U(1)_a$
  - v^3 type : cannot gauge $U(1)_v$
  - Mixed type : cannot gauge both $U(1)_a \times U(1)_v$

Global symmetry after gauging

- But with (mixed) anomaly, the Ward identity become

$$\partial_\mu \langle j_a^\mu \rangle = \kappa_{a^2 v} (da)_{\mu\nu} \langle J^{\mu\nu} \rangle + \kappa_{a v^2} \epsilon^{\mu\nu\rho\sigma} \langle J_{\mu\nu} \rangle \langle J_{\rho\sigma} \rangle$$

  - Can no longer captured by symmetry under $\begin{array}{l} a \to a + d\lambda \\ b \to b + d\Lambda \end{array}$

- For one of these two anomaly, we know that

$$Z[a_\mu, b_{\mu\nu}] = \left\langle \exp\left[i \int d^{3+1}x \sqrt{-g} \left(j_a^\mu a_\mu + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

with $a \to a + d\lambda \qquad b \to b + d\Lambda + \hat{\kappa}(da)\lambda$

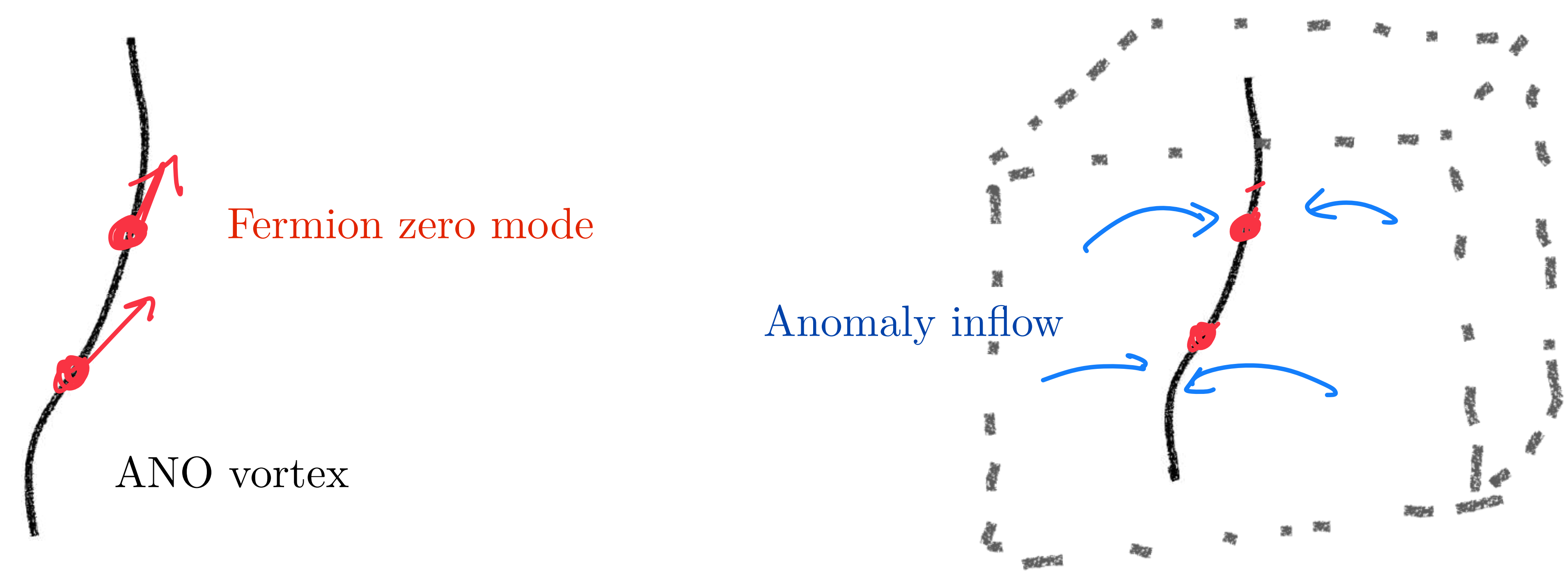
Give the desirable ward identity. $b_{\mu\nu} \sim$ GS 2-form

$$\partial_\mu \langle j_a^\mu \rangle = \hat{\kappa} \langle J^{\mu\nu} \rangle (da)_{\mu\nu}$$

This structure is called 2-group

(Coherent) 2-group global symmetry
Physical example(s)

- Abrikosov-Nielsen-Olesen vortex with in superconductor QED with a certain anomaly



2d massless fermion is anomalous

Total system is anomaly free

- These zero modes affect macroscopic descriptions

2-GROUP AS A GENUINE SYMMETRY

* But with (mixed) anomaly, the Ward identity become

$$\partial\langle j_a^\mu\rangle = \kappa_{a^2\nu}(da)_{\mu\nu}\langle J^{\mu\nu}\rangle + \kappa_{a\nu^2}\epsilon^{\mu\nu\rho\sigma}\langle J_{\mu\nu}\rangle\langle J_{\rho\sigma}\rangle$$

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Recap and some unanswered questions

* Gaging is a dramatic process

$$|q_{points1}, q_{points2}\rangle \quad \Rightarrow \quad |q_{points1}, Q_{lines}\rangle$$

Quantum number q, Q are independent if (ungauge) $U(1) \times U(1)$ are not anomalous

* How to get 2-group from gauging non-anomalous subgroup

* Let's treat it as a genuine global symmetry (bypassing the gauging procedure)

$$Z[a_\mu, b_{\mu\nu}] = \left\langle \exp\left[i \int d^{3+1}x \sqrt{-g} \left(j^\mu_a a_\mu + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

$$a \to a + d\lambda$$

$$b \to b + d\Lambda + \hat{\kappa}(da)\lambda$$

$$\partial_\mu \langle j^\mu_a \rangle = \hat{\kappa} \langle J^{\mu\nu} \rangle (da)_{\mu\nu}$$

Hydrodynamics of 2-group ?

If we think of hydro as dynamics of Noether currents

- 2-group is a genuine symmetry structure so why not?
  - 2-group found in various interesting QFTs

...; Baez & Lauda '03;
Kapustin & Thorngren '13; Barkeshli et.al. '14
Sharpe '15; Tachikawa '17
Cordova, Dumitrescu & Intriligator '18
Delcamp & Tiwari 18; Benini, Cordova & Hsin '18, Hsin & Lam '20 ....

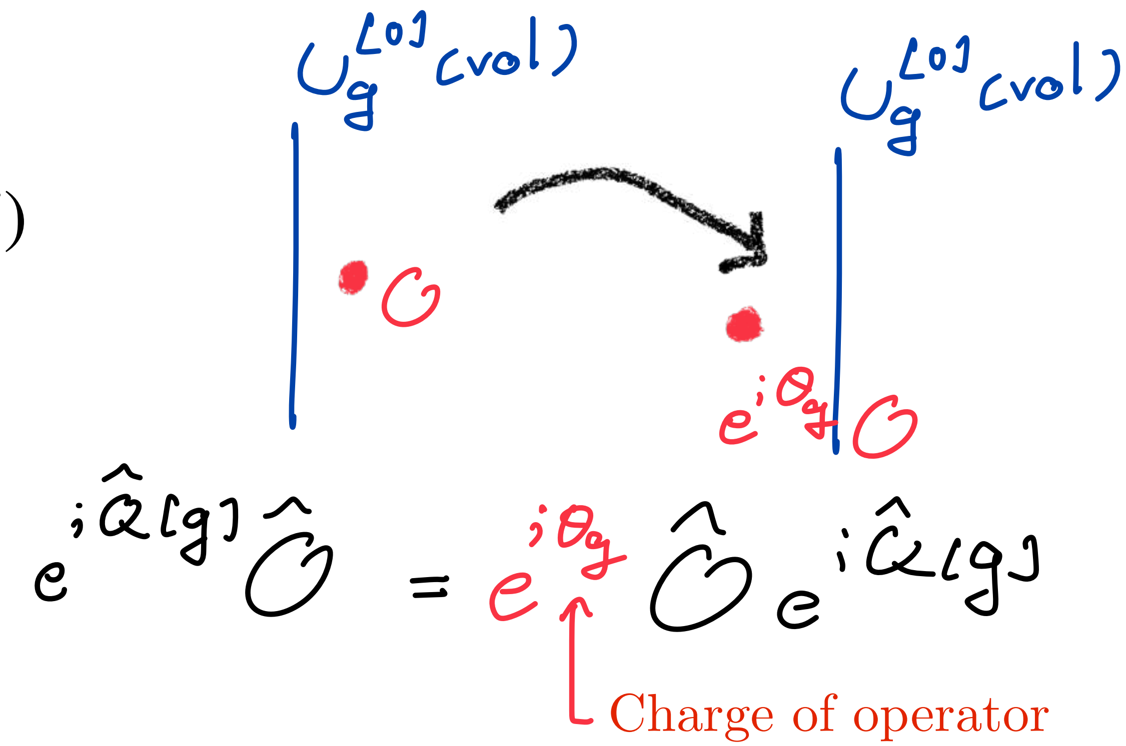
- An interesting way to systematically extend hydrodynamic framework beyond ordinary continuous global symmetry
- Toy model for chiral MHD

Summary of the results

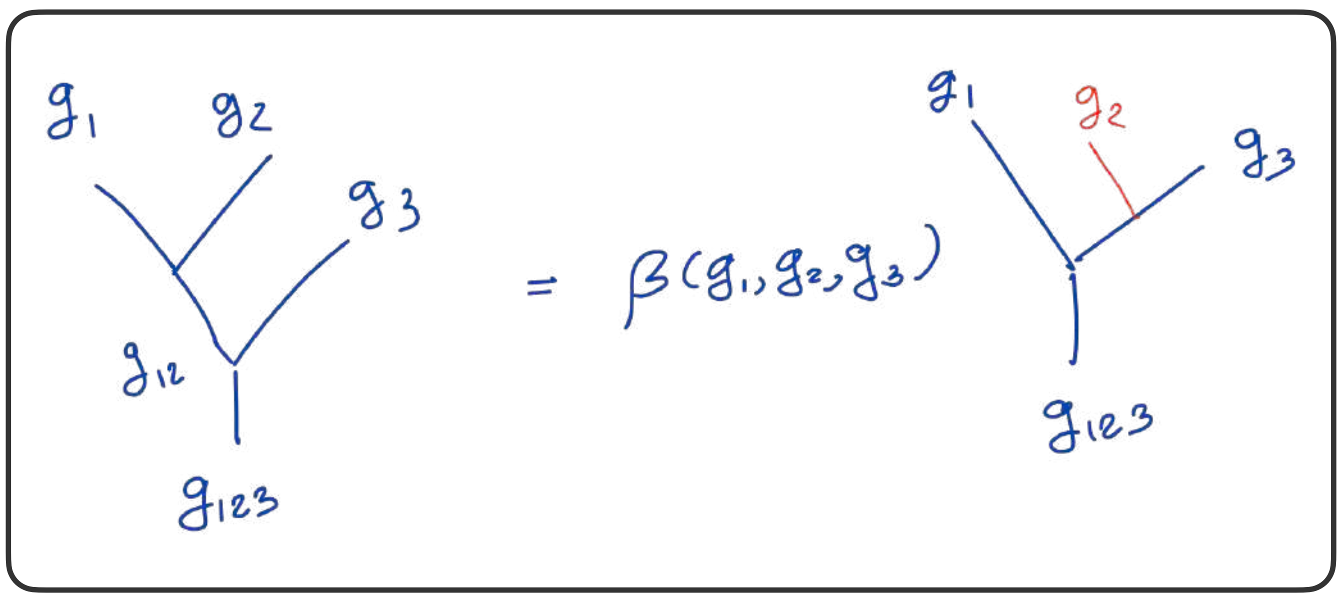
- ✻ New phenomena similar to (continuous) anomaly induce transport in 1+d dim (CME, CVE, C....)
  - But the system is anomaly free and can lives in higher dimensions
- ✻ A procedure to obtain hydrodynamics equations from 2-group background fields
- ✻ Holographic dictionary & Consistency with 2-group gauge theory in the bulk

(Coherent) 2-group global symmetry

* Consist of a group G generated by $U_g^{[0]}(vol)$
  and Abelian group $\mathscr{A}$ generated by $U_a^{[1]}(surf)$



* Things works as usual, except associativity



$$g_i \in G$$

$$\beta : G \times G \times G \to \mathscr{A}$$

$$[\beta] \in H^3(BG, \mathscr{A})$$

$$(e^{iQ[g_1]}e^{iQ[g_2]})e^{iQ[g_3]}|\Psi\rangle = e^{iQ[g_1]}(e^{iQ[g_2]}e^{iQ[g_3]})\beta[g_1, g_2, g_3]|\Psi\rangle$$

See e.g. Kapustin & Thorngren '13;
Benini, Cordova & Hsin '18
Cordova's talk at strings 2020

(Coherent) 2-group global symmetry

$g_i \in G \qquad \beta : G \times G \times G \to \mathscr{A}$

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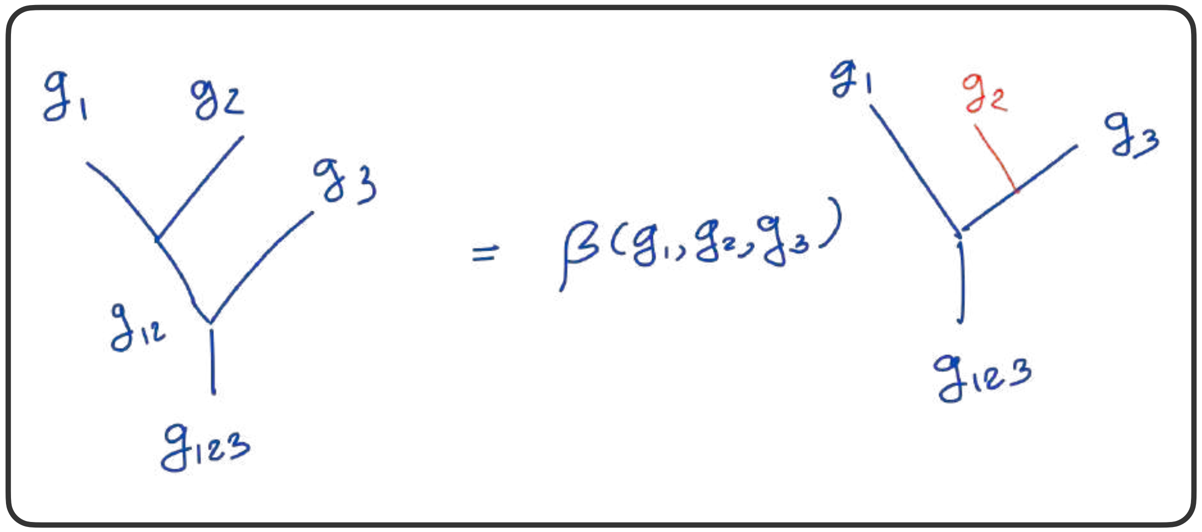


* Note that the state $|\Psi\rangle$ characterized by "2" quantum numbers $(G, \mathscr{A})$

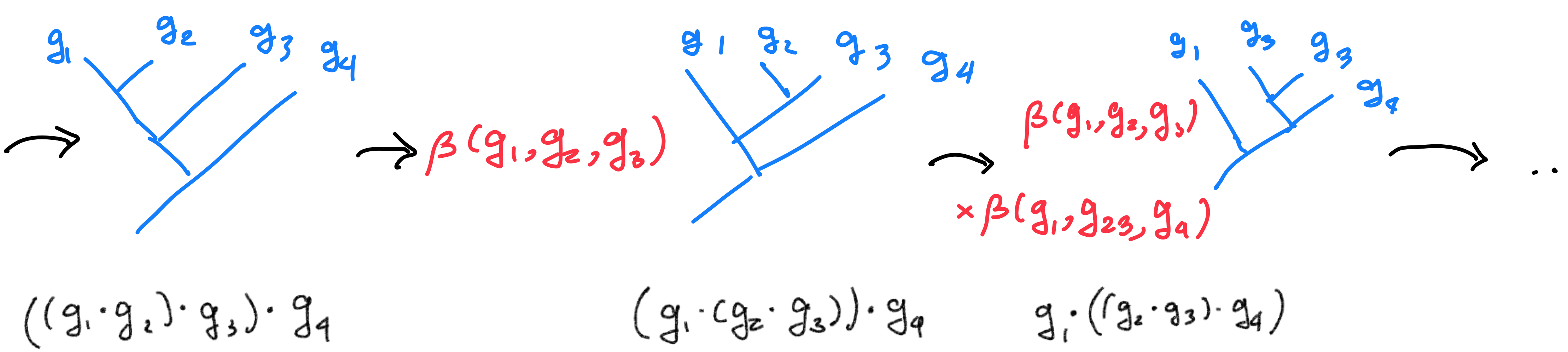
See e.g. Kapustin & Thorngren '13;

Benini, Cordova & Hsin ' 18

Cordova's talk at strings 2020

$$(e^{iQ[g_1]}e^{iQ[g_2]})e^{iQ[g_3]}|\Psi\rangle = e^{iQ[g_1]}(e^{iQ[g_2]}e^{iQ[g_3]})\beta[g_1, g_2, g_3]|\Psi\rangle$$

* Postnikov class β must satisfy the pentagon identity



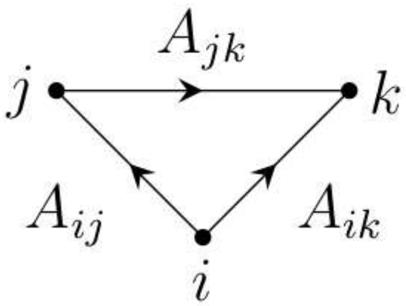
(Coherent) 2-group global symmetry

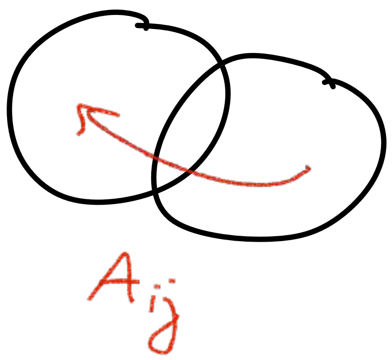
Benini, Cordova & Hsin ' 18

* Equivalently, via transition function (similar to what used in Čech cohology)

$$A_{ij}A_{jk} = A_{ik}$$

$$\int_{\xi_2} F \in \mathbb{Z}$$

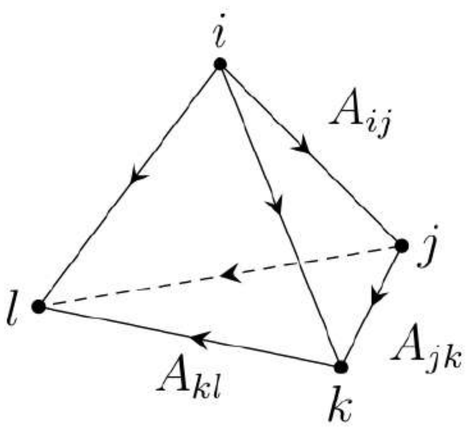




* With additional structure

$$B_{jkl} - B_{ikl} + B_{ijl} - B_{ijk} = \beta(A_{ij}, A_{jk}, A_{kl})$$

$$\int_{\xi_3} (db - \hat{\kappa} a \wedge da) \in \mathbb{Z}$$



$$A \in C^1(\mathcal{M}, G)$$

$$B \in C^2(\mathcal{M}, \mathcal{A})$$

$$g_i \in G$$

$$\beta : G \times G \times G \to \mathcal{A}$$

$$[\beta] \in H^3(BG, \mathcal{A})$$

* More definitions from category p.o.v.

Sharpe '15, Baez & Huerta '10 (review), Baez & Lauda '03

Building Hydrodynamics for 2-group global symmetry

* (No)Entropy production constraints

* Geometric realisation of equilibrium partition function

* Effective action approach + holographic deconstruction

Ordinary fluid with U(1) global symmetry

* Conserved currents T^{tt}, T^{ti}, j^t with conjugates T, u^i, μ_a

$$j^\mu = \rho_a u^\mu + \mathcal{O}(\partial)$$
$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu} + \mathcal{O}(\partial)$$

$$\partial_\mu \langle j^\mu \rangle = 0\,,$$
$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

* One can check that (on shell) entropy current is constant

$$s^\mu = p u^\mu - \left(\frac{u_\nu}{T}\right) T^{\mu\nu} - \left(\frac{\mu_a}{T}\right) j^\mu$$
$$= s u^\mu$$

Where $\partial_\mu s^\mu = 0$ ⟶

$$dp = s dT + \rho_a d\mu_a\,,$$
$$\varepsilon + p = sT + \mu_a \rho_a$$

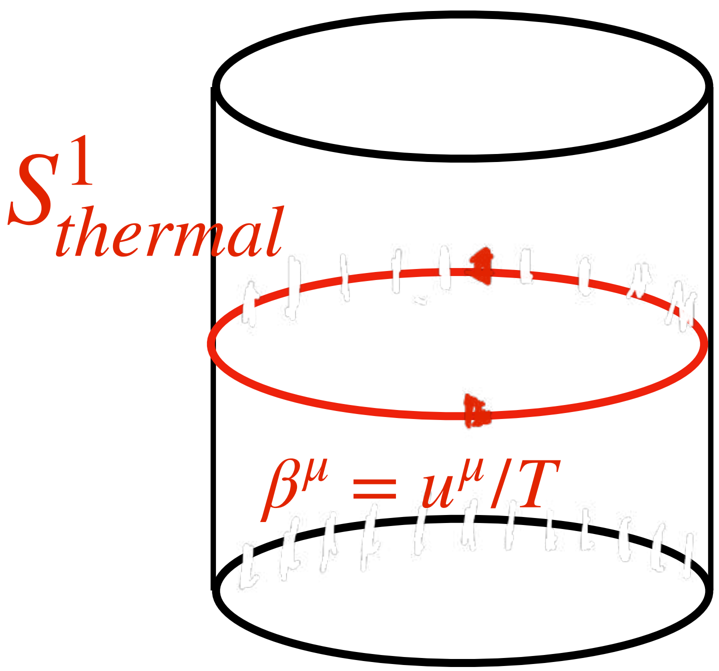
Geometric interpretation: Ordinary fluid case

* All of them can be written in terms of $\beta^\mu = u^\mu/T$ and μ_a/T

$$\langle j^\mu \rangle = \rho_a u^\mu + \mathcal{O}(\partial)$$

c.f. Landau & Lifshitz book

$$\langle T^{\mu\nu} \rangle = (\varepsilon + p) u^\mu u^\nu + p g^{\mu\nu} + \mathcal{O}(\partial)$$



$1/T \sim$ **size of thermal cycle**

$$\mu_a/T \sim \ln \mathcal{P} e^{i\int_{S^1} a} \sim \int a_\mu u^\mu d\tau$$
$\sim$ **background U(1) holonomy**

Jensen et.al. '12

Banerjee et.al. '12

Haehl, Loganayagam & Rangamani '15

* Constitutive relations obtained via $\log Z \sim p\left(T = 1/\sqrt{-\beta^\mu \beta_\mu}, \mu_a\right)$

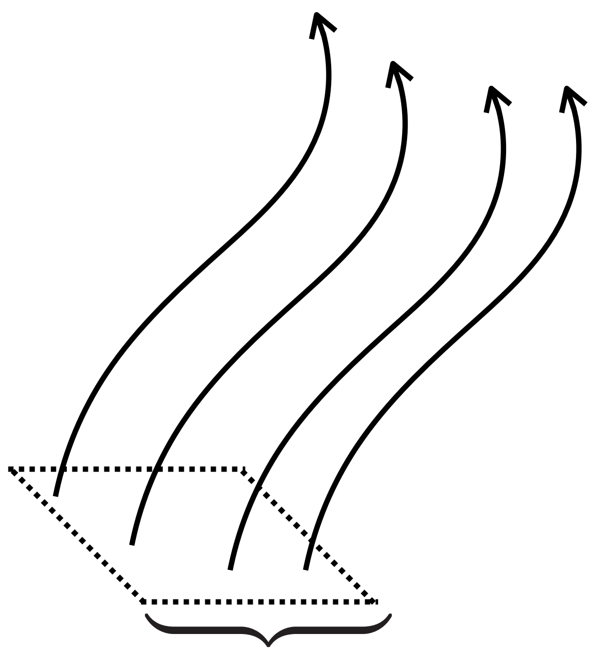
Geometric interpretation: fluid + strings

....

Emparan, Harmark, Niarchos & Obers '09
Caldarelli, Emparan & van Pol '11
Schubring '14
Grozdanov, Hofman & Iqbal '16

✻ All of them can be written in terms of $\beta^\mu = u^\mu/T$ and $\mu_b h_\mu/T$

$$\langle J^{\mu\nu}\rangle = \rho_b(u^\mu h^\nu - u^\nu h^\mu) + \mathcal{O}(\partial)$$
$$\langle T^{\mu\nu}\rangle = (\varepsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \mu_b \rho_b h^\mu h^\nu + \mathcal{O}(\partial)$$



$1/T$ ~ size of thermal cycle

$$\mu_b \sim \ln \mathcal{P} \exp\left(i\int_{T^2} b\right) \sim \int (b_{\mu\nu}u^\mu h^\nu)\, d^2\sigma$$

~ **Wilson surface**

Or smarter definition without torus

Armas & Jain '18
Glorioso & Son '18

$$\mu_b \quad \Rightarrow \quad \mu_b h_\mu$$

✻ Constitutive relations obtained via $\log Z \sim p\left(T = 1/\sqrt{-\beta^\mu\beta_\mu}, \mu_b\right)$

Relations to ideal MHD

- To related to traditional MHD, take $u^\mu = (1, V^i)$, $h^\mu = (0, B^i/\rho_b)$

- The electric field is $E^i = \frac{1}{2}\epsilon^{ijk}J_{jk} = -(V \times B)^i$

  Ideal Ohm's law

  $$E + V \times B = \frac{j}{\sigma} \to 0$$

- Ward identity $\partial_\mu J^{\mu\nu} = 0$, encodes

  $$\partial_i B^i = 0$$

  Gauss' law

  $$\partial_t B^i + (\nabla \times E)^i = 0$$

  Faraday's law

- Assuming $p = \ldots + \frac{1}{2}\mu_b^2$ so that $\mu_b/\rho_b \sim$ const

$$\varepsilon(\partial_t + V \cdot \partial)V^i = -\partial^i p + \big((\partial \times B) \times B\big)^i$$

Euler + Lorentz Force + Ampere's law $\quad j = \nabla \times B - \partial_t E$ ($\partial_t E \approx 0$)

We can even patch them together ?

$$Z[a_\mu, b_{\mu\nu}] = \left\langle \exp\left[-\int d^4x\sqrt{-g}\left(j^\mu a_\mu + \frac{1}{2}J^{\mu\nu}b_{\mu\nu}\right)\right]\right\rangle$$

$$\log Z \sim p(T, \mu_a, \mu_b)$$

$$\langle T^{\mu\nu}\rangle = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \mu_b\rho_b h^\mu h^\nu + \mathcal{O}(\partial)$$

$$\langle J^{\mu\nu}\rangle = \rho_b\left(u^\mu h^\nu - u^\nu h^\mu\right) + \mathcal{O}(\partial)$$

$$\langle j^\mu\rangle = \rho_a u^\mu + \mathcal{O}(\partial)$$

* This would be true if the theory has $U(1)^{[0]} \times U(1)^{[1]}$ symmetry

Now, we are ready to combine them into 2-group

$$Z[a_\mu, b_{\mu\nu}] = \left\langle \exp\left[i \int d^{3+1}x \sqrt{-g}\, \left(j_a^\mu a_\mu + \frac{1}{2} J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

$$a \to a + d\lambda \qquad b \to b + d\Lambda + \hat{\kappa}(da)\lambda$$

What went wrong in the naive picture (1): Entropy production can be negative.

- If we still insist to have everything written in terms of $\{T, \mu_a, \mu_b, u^\mu, h^\mu\}$ as in $U(1)_a \times U(1)_b$ case, the entropy current

$$
\begin{aligned}
s^\mu &= p u^\mu - \left(\frac{u_\nu}{T}\right) T^{\mu\nu} - \left(\frac{\mu_a}{T}\right) j^\mu - \left(\frac{\mu_b h_\nu}{T}\right) J^{\mu\nu} + \tilde{s}^\mu \\
&= s u^\mu
\end{aligned}
$$

will not be constant, even at ideal level! $\quad \partial_\mu s^\mu \neq 0$

- Similar situation happens in anomalous fluid as $\partial_\mu j^\mu \neq 0$

Son & Surowka '09 ;

WHAT WENT WRONG IN THE NAIVE PICTURE (2): NO WILSON SURFACE TO DEFINE CHEMICAL POTENTIAL

* Previously, we said that string chemical potential is

$$\mu_b \sim \int_{T^2} b$$

Invariant under $a \to a + d\lambda$, $b \to b + d\Lambda$

But now

$$\int_{T^2} b \to \int_{T^2} b + \hat{\kappa} \int_{T^2} \lambda(da)$$

chem pot transform like anomalous partition function

* If "μ_b" is not invariant, we can't include it in Z

  - The string density $\rho_b = \dfrac{\partial p}{\partial \mu_b}$ always zero ?
  - **Or new definition of chemical potential ?**

2-GROUP CHEMICAL POTENTIAL

* Previously, we said that string chemical potential is

$$\mu_b \sim \int_{T^2} b$$

But now

$$\int_{T^2} b \to \int_{T^2} b + \hat{\kappa} \int_{T^2} \lambda(da)$$

Invariant under $a \to a + d\lambda$, $b \to b + d\Lambda$

chem pot transform like anomalous partition function

* Intuitively, we would want

$$\mu_b \sim \int_{T^2} b + S_{WZ}[\phi, a]$$

* Such that $W[a, b, \phi, \varphi]$ preserve the same global symmetry

Dubovsky, Hui, Nicolis '11; Haehl-Logan-Rangamani '13

Delacretaz & Glorioso '20

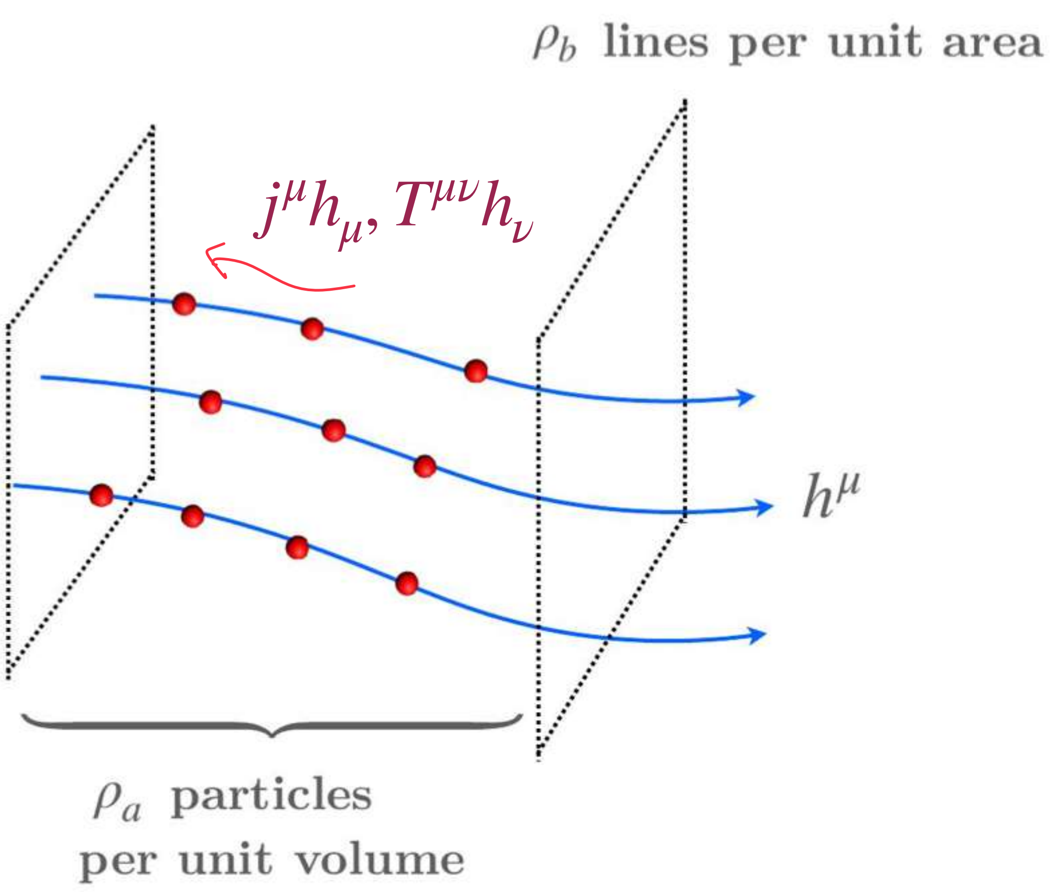
Observable consequences

$$\langle T^{\mu\nu}\rangle = (\varepsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \mu_b \rho_b h^\mu h^\nu - \hat{\kappa}\mu_a^2 \rho_b (u^\mu h^\nu + u^\nu h^\mu)\,,$$

$$\langle J^{\mu\nu}\rangle = \rho_b (u^\mu h^\nu - u^\nu h^\mu)\,,$$

$$\langle j^\mu\rangle = \rho_a u^\mu - 2\hat{\kappa}\mu_a \rho_b h^\mu - \hat{\kappa}\langle J^{\mu\nu}\rangle a_\nu$$

* New terms that fixed entropy production coming from WZ term



Observable consequences

* It alters the currents 1-pt function

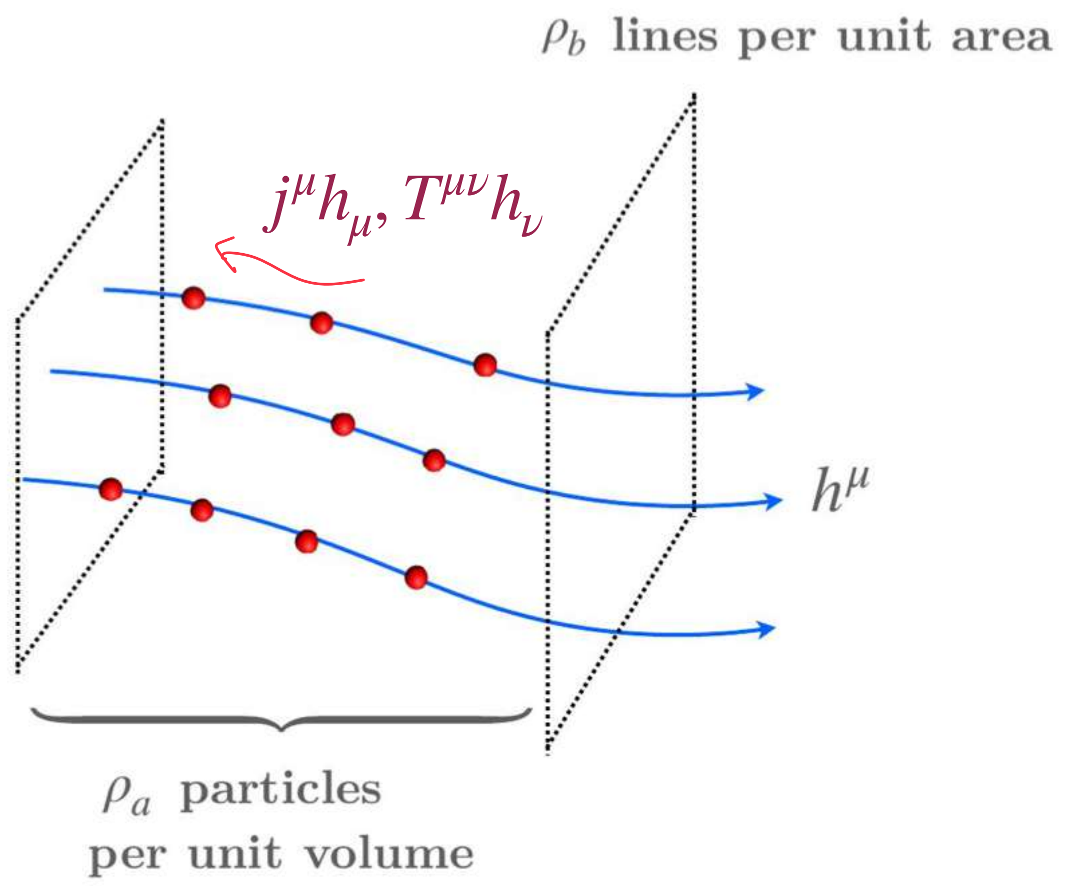
$$\langle T^{\mu\nu}\rangle = (\varepsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \mu_b \rho_b h^\mu h^\nu - \hat{\kappa}\mu_a^2 \rho_b (u^\mu h^\nu + u^\nu h^\mu)\,,$$
$$\langle J^{\mu\nu}\rangle = \rho_b (u^\mu h^\nu - u^\nu h^\mu)\,,$$
$$\langle j^\mu\rangle = \rho_a u^\mu - 2\hat{\kappa}\mu_a \rho_b h^\mu - \hat{\kappa}\langle J^{\mu\nu}\rangle a_\nu$$

* Change speed of transverse sound

$$\omega_\perp = \left(-\frac{\hat{\kappa}\mu_a^2\rho_b}{\varepsilon+p} \pm \sqrt{\mathcal{V}_A^2 + \left(\frac{\hat{\kappa}\mu_a^2\rho_b}{\varepsilon+p}\right)^2}\right) q_z\,, \qquad \mathcal{V}_A^2 = \frac{\mu_b\rho_b}{\varepsilon+p}$$

* Additional chiral sound

$$\omega_\parallel = -\frac{\hat{\kappa}\rho_b}{\chi_{aa}} q_z\,.$$



2-GROUP CHEMICAL POTENTIAL

* Previously, we said that string chemical potential is

$$\mu_b h_\mu = u^\nu \left(b + d\varphi + \hat{\kappa}\phi(da)\right)_{\nu\mu} - \hat{\kappa}(u^\nu A_\nu)A_\mu$$

* It alters the currents 1-pt function

$$\langle T^{\mu\nu}\rangle = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \mu_b\rho_b h^\mu h^\nu - \hat{\kappa}\mu_a^2\rho_b(u^\mu h^\nu + u^\nu h^\mu)\,,$$

$$\langle J^{\mu\nu}\rangle = \rho_b(u^\mu h^\nu - u^\nu h^\mu)\,,$$

$$\langle j^\mu\rangle = \rho_a u^\mu - 2\hat{\kappa}\mu_a\rho_b h^\mu - \hat{\kappa}\langle J^{\mu\nu}\rangle a_\nu$$

Holographic dual

* Holographic action

$$\mathcal{H} = d\mathcal{B} - \hat{\kappa}\mathcal{A}\wedge d\mathcal{A}$$

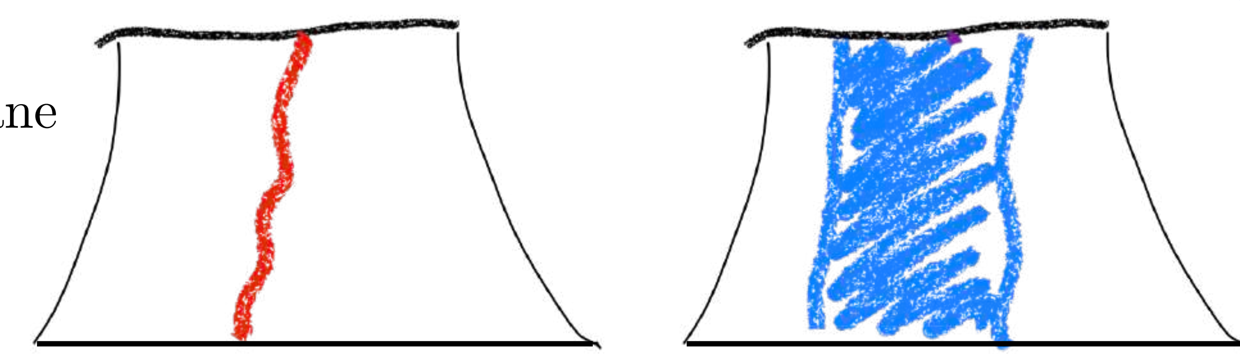
$$S_{bulk} = -\int d^{d+2}X\sqrt{-G}\left(\frac{1}{4}\mathcal{F}_{ab}\mathcal{F}_{ab} + \frac{1}{6}\mathcal{H}_{abc}\mathcal{H}^{abc}\right)$$

Cordova, Dumitrescu & Intrilligator '18

* Dictionary: background field ⇔ Dynamical gauge field in higher dim

$$\mathscr{A}_\mu(r \to \infty) \sim a_\mu$$
$$\mathscr{B}_{\mu\nu}(r \to \infty) \sim b_{\mu\nu}$$

Black brane horizon



$r \to \infty$

$\phi \sim \int dr\, \mathscr{A}_r$

$\int dx^\mu \varphi_\mu \sim \int dx^\mu \int dr\, \mathscr{B}_{r\mu}$

Quainormal modes = poles in $\langle j^\mu j^\nu \rangle = \dfrac{\delta^2 S_{grav}}{\delta a_\mu \delta a_\nu}$

Holographic dual

- Holographic deconstruction of

$$\mathcal{H} = d\mathcal{B} - \hat{\kappa}\mathcal{A}\wedge d\mathcal{A}$$

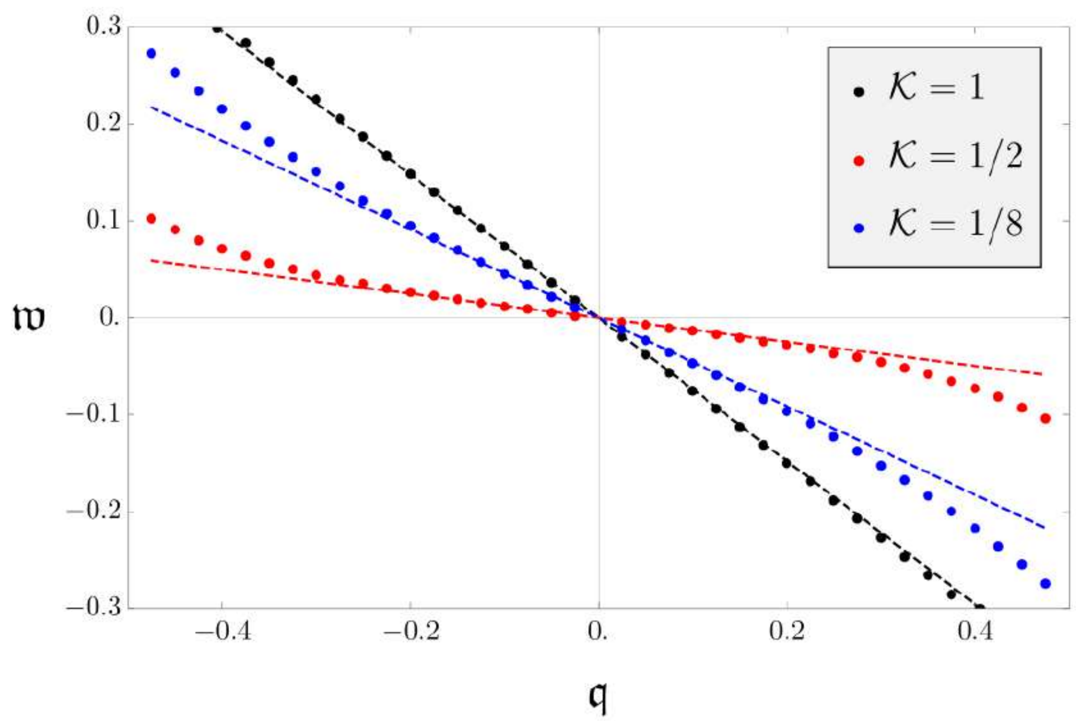
$$S_{bulk} = -\int d^{d+2}X\sqrt{-G}\left(\frac{1}{4}\mathcal{F}_{ab}\mathcal{F}_{ab} + \frac{1}{6}\mathcal{H}_{abc}\mathcal{H}^{abc}\right)$$

Cordova, Dumitrescu & Intrilligator '18

$$\phi(x^\mu) = \int_{r_h}^{\infty} dr'\mathcal{A}_r\,, \qquad \varphi_\mu(x^\mu) = \int_{r_h}^{\infty} dr'\left(\mathcal{B}_{r\mu} - \phi\,\mathcal{F}_{r\mu}\right)$$

$$\mathcal{K} := \hat{\kappa}\rho_b/r_h^2$$

- Additional chiral sound encoded in QNM



Without $\hat{\kappa}$: $\langle j^x(-q_x)j^x(q_x)\rangle \sim \dfrac{1}{\omega + iDq_x^2}$

Summary of the results

* New phenomena similar to (continuous) anomaly induce transport in 1+d dim (CME, CVE, C....)
  - But the system is anomaly free and can lives in higher/ odd dimensions
  - Anomalous transport occur at order (n-1) for $d+1 = 2n$ in derivative expansion. 2-group always appears at 0th order
  - Only rely on global symmetry & fully covariant

Reasonable future direction?

* More interesting QFT has discrete 1-form symmetry

Categorical symmetry : Ji & Wen '19

Topological phases : Kapustin & Thorngren '14 ; Delcamp & Tiwari '19

Charn-Simons + matter : Benini, Cordova & Hsin '18

QCD & colour-flavour-center symmetry : Cherman, Sen, Unsal, Wagman & Yaffe ;

* Modes that localized on sub-manifold like fracton?

* Even more weird symmetry structure?
  - Higher-group in QCD and axion QED ?

    Tanizaki & Unsal '19

    Hidaka & Nita '20

  - 'Categorical' symmetry ?

    Bhardwaj & Tachikawa '17 ;

    Tachikawa '17;

  - Interplay with holography/hydro and discrete anomaly?

Even more directions ?

* How to deal with the other anomaly ?
  Chiral MHD and astrophysical application ?

* Instead of U(1) conserved flux, we can look at discrete symmetry ?

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* Holographic dual of discrete higher group ?

Thank you very much!