The phase of UV complete theories at high temperature

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Introduction

We all know that heated systems typically become less ordered

In other words one expects that the ground state is more symmetric at high temperature than at low temperature

Or at least it is not less symmetric

But not always!

Increasing the temperature sometimes the opposite happens

Two known examples

- Rochelle salt
- some liquid crystals (SmC^*)

In both cases two critical temperatures: $T_{c1} < T_{c2}$

Outside the interval

$T < T_{c1}$ and $T > T_{c2}$ more symmetric phase (less order)

In between $T_{c1} < T < T_{c2}$ less symmetric phase (more ordered)

Phase transition at higher T_{c2} as we are used: heating the system we get more symmetrical vacuum

Phase transition at T_{c1} counter-intuitive: heating the system we get less symmetrical vacuum

How do we describe such behaviours in field theory?

Field theory

Toy model

System's degree of freedom scalar field ϕ

symmetry of the Lagrangian Z_2: $\phi \to -\phi$

Potential

$$V(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$\lambda > 0$ for the potential to be bounded from below

For $m^2 > 0$

$$\frac{\partial V}{\partial \phi} = 0 \to \langle\phi\rangle = \pm\sqrt{\frac{m^2}{\lambda}} \neq 0$$

the ground state has a spontaneously broken Z_2 symmetry

Vacuum is not symmetric under Z_2

This was at $T = 0$

What happens at high T ($\gg m$)?

Field theory tells us

Weinberg '74

$$\Delta V_T = \frac{T^2}{24}\frac{\partial^2 V}{\partial \phi^2} \to \frac{T^2}{8}\lambda\phi^2$$

i.e. one adds a thermal mass

$$m^2(T) = \frac{\lambda}{4}T^2 > 0$$

Increasing the temperature we arrive at the phase transition when

$$-m^2 + m^2(T_c) = 0 \to T_c^2 \approx \frac{m^2}{\lambda}$$

At high $T \gg T_c$ symmetry restored

At low $T \ll T_c$ symmetry broken

Boundedness of the potential ($\lambda > 0$) leads to symmetry restoration at high temperature

As expected

Only the quartic important for the conclusion (mass terms irrelevant in the UV)

How can one describe the opposite transition (inverse symmetry breaking or symmetry non-restoration) ?

At least 2 fields

Weinberg '74, Mohapatra, Senjanović '79

$$V = \frac{\lambda_1}{4}\phi_1^4 + \frac{\lambda_2}{4}\phi_2^4 - \frac{\lambda}{2}\phi_1^2\phi_2^2$$

two discrete symmetries

$Z_2 \times Z_2$: $\phi_1 \to -\phi_1$ and (independent) $\phi_2 \to -\phi_2$

Boundedness from below of V gives constraints on parameters:

$$V = \frac{1}{4}\begin{pmatrix}\phi_1^2 & \phi_2^2\end{pmatrix}\begin{pmatrix}\lambda_1 & -\lambda \\ -\lambda & \lambda_2\end{pmatrix}\begin{pmatrix}\phi_1^2 \\ \phi_2^2\end{pmatrix} > -\infty$$

$$\to \quad \lambda_{1,2} > 0 \quad , \quad \lambda_1\lambda_2 > \lambda^2$$

Thermal mass (matrix) at high temperature

$$\Delta V_T = \frac{T^2}{48}\frac{\partial^2 V}{\partial\phi_k\partial\phi_k} \equiv \frac{1}{2} m_{ij}^2(T)\phi_i\phi_j$$

$$m_{ij}^2(T) \to \frac{T^2}{24}\begin{pmatrix} 3\lambda_1 - \lambda & 0 \\ 0 & 3\lambda_2 - \lambda \end{pmatrix}$$

One eigenvalue can be negative (say 11)

$$3\lambda_1 - \lambda < 0$$

providing

$$\lambda_2 > \frac{\lambda^2}{\lambda_1}$$

and so the second eigenvalue (22) will be positive

One eigenvalue of $m^2(T)$ negative (breaks symmetry)

one positive (preserves symmetry)

$$\underbrace{Z_2 \times Z_2}_{\text{symmetry of } V} \quad \rightarrow \quad \underbrace{Z_2}_{\text{symmetry of vacuum}}$$

At arbitrary high temperature!

Example of symmetry non-restoration at high temperature

This mechanism (symmetry non-restoration at high temperature) used in many cosmological context

- domain wall problem

Dvali,Senjanović '95, Dvali,Melfo,Senjanović '96, ...

- monopole problem

Langacker,Pi '80, Dvali,Melfo,Senjanović '95, ...

- false vacuum problem

Example: inflation could have been also before GUT symmetry breaking scale because GUT symmetry may have never been restored

→ no dangerous monopole production during phase transition (which is no more there)

Consistency

Here we will be interested in the following question: is the above true at arbitrary high temperature? Or is it like Rochelle salt, with symmetry eventually restoring?

In many application this answer has no real effect: Planck scale will make us stop thinking what is above (quantum gravity, string theory ?)

But

- gravity could be soft
- good to know what field theory allows and what not

So here we are interested in: Does field theory permits symmetry non-restoration at arbitrary high T?

What could go wrong?

Is the Weinberg example a UV complete theory?

1-loop RGE of a scalar theory (I use the convention $4\pi = 1$)

$$\mu \frac{d\lambda}{d\mu} = +3\lambda^2$$

Solution:

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - 3\lambda(\mu_0) \log\left(\frac{\mu}{\mu_0}\right)}$$

has a Landau pole ($\lambda \to \infty$) at

$$\mu = \mu_0 \exp\left(\frac{1}{3\lambda(\mu_0)}\right)$$

$\rightarrow$ pure scalar theories not UV complete, they have a cutoff (Landau pole)

For this conclusion crucial + sign in RGE

To make theories UV complete (well defined) one should add more stuff (fermions, gauge bosons)

Two possibilities at short distances (large energies = UV)

- asymptotically free theory (all couplings go to zero in the UV, perturbative)
- asymptotically safe theory (couplings go to finite values in the UV, possibly perturbative or non-perturbative)

Here we will first concentrate on asymptotically free theories

Asymptotically free theories

Simplest asymptotically free theory : pure Yang-Mills

Gross, Wilczek '73, Politzer '73

take $SU(N_c)$ gauge group; RGE for gauge coupling g:

$$\mu \frac{dg^2}{d\mu} = -2b_0 g^4 \quad , \quad b_0 = \frac{11}{3} N_c$$

solution:

$$g^2(\mu) = \frac{1}{2b_0 \log(\mu/\mu_0)}$$

Theory has a IR cutoff (below it confinement?) but is well defined for arbitrary large μ, coupling goes to zero in UV ($\mu \to \infty$)

For this conclusion crucial $b_0 > 0$

To study symmetry breaking we add a Higgs (ϕ)

- potential $V(\phi)$
- Yukawa couplings with fermions

$$\mathcal{L}_{Yukawa} = \phi\bar{\psi}y\psi$$

The thermal mass gets new contributions:

$$m^2(T) \sim T^2 \left(V''(\phi) + y^2 + C_2 g^2\right)$$

$C_2 \ldots$ second Casimir of the gauge group representation of ϕ

Notice the positive signs of the new contributions

- harder to get symmetry non-restoration
- only $V''(\phi)$ can help (at least 2 scalars)

But more than that:

- V, y and g must satisfy RGEs

The fixed flow solution of RGEs

We are interested in the thermal mass at high T on the solution of the RGEs.

In general this very difficult: RGEs are coupled non-linear first order differential equations for the Lagrangian parameters $p_i = g^2, y^2, \lambda, \ldots$

$$\mu \frac{dp_i}{d\mu} = \sum_j c_{ij} p_i p_j + \text{ higher loops}$$

$c_{ij} \ldots$ model dependent numbers

Fortunately we are interested only in

- solutions in asymptotic regime $\mu \sim T \to \infty$
- only those that have a chance to give $m_T^2 < 0$

Then it is enough to look for *fixed flow* solutions (ansatz)

$$p_i = \frac{\tilde{p}_i}{\log \mu}$$

$\tilde{p}_i \ldots$ numbers which satisfy

$$-\tilde{p}_i = \sum_j c_{ij} \tilde{p}_i \tilde{p}_j$$

Set of coupled non-linear *algebraic equations*; still difficult in general but much easier than differential equations

The **proof** is simple (schematic - all positive coefficients $\to 1$):

1. to avoid the Landau pole of the Higgs couplings we should add gauge and Yukawa couplings

$$\frac{d\lambda}{dt} = \underbrace{\lambda^2 + \lambda y^2 + g^4}_{\text{make } \lambda \text{ increase}} \; \underbrace{-\lambda g^2 - y^4}_{\text{make } \lambda \text{ decrease}}$$

with $t = \log\mu$

2. gauge coupling goes like $1/t$

$$\frac{dg^2}{dt} = -g^4$$

3. to avoid the Landau pole for the Yukawa couplings we should add the gauge coupling

$$\frac{dy^2}{dt} = y^4 - y^2 g^2$$

Yukawa coupling goes at large t as $1/t^k$ with $k \geq 1$

4. Higgs coupling cannot dominate over gauge and/or Yukawa couplings

$$g^2 \sim 1/t \quad , \quad y^2 \sim 1/t$$

$\rightarrow$ Higgs couplings cannot decrease slower than $1/t$

$$\rightarrow \lambda \sim 1/t$$

QED

Two singlet Higgses

Let's construct a minimal UV complete model

- at least two scalars needed
- to minimise the thermal mass take $C_2 = 0$
  $\rightarrow$ gauge singlet scalars
- then only Yukawa can avoid Landau pole $y \neq 0$

Potential

$$V = \frac{\lambda_1}{4}\phi_1^2 + \frac{\lambda_2}{4}\phi_2^2 - \frac{\lambda}{2}\phi_1^2\phi_2^2$$

The difference with Weinberg example is that we add

$$\mathcal{L}_{Yukawa} = y_1\phi_1 \sum_{j=1}^{N_1} \overline{\psi}_{1j}\psi_{1j} + y_2\phi_2 \sum_{j=1}^{N_2} \overline{\psi}_{2j}\psi_{2j}$$

ψ_{1j} ... only coupled to ϕ_1

ψ_{2j} ... only coupled to ϕ_2

This needed to have a discrete $Z_2 \times Z_2$ symmetry:

$$\phi_a \to -\phi_a \quad , \quad \psi_{aj} \to i\gamma_5\psi_{aj} \quad , \quad a = 1, 2$$

ψ_{1j} and ψ_{2j} in the fundamental representation of $SU(N_c)$

So in principle one should

- write down all the RGE
- find all the fixed flow solutions
- check whether they lead or not to (at least one) negative eigenvalue of m_T^2

Fortunately all this is not needed in our case

All we need is the fixed flow ($p_i = \tilde{p}_i/t$) RGE for $\tilde{\lambda}_1$:

$$-\tilde{\lambda}_1 = 18\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 + 8N_1N_c\tilde{\lambda}_1\tilde{y}_1^2 - 8N_1N_c\tilde{y}_1^4$$

Is this compatible with

$$m_1^2(T) = \frac{T^2}{12\log T}\left(3\tilde{\lambda}_1 - \tilde{\lambda} + 2N_1N_c\tilde{y}_1^2\right) < 0 \quad ?$$

The answer is no, for any choice of N_1 and N_c.

Proof:

On the one side to get $m_1^2(T) \propto \left(3\tilde{\lambda}_1 - \tilde{\lambda} + 2N_1 N_c \tilde{y}_1^2\right) < 0$ we need

$$\tilde{\lambda} - 2N_1 N_c \tilde{y}_1^2 > 3\tilde{\lambda}_1 > 0 \qquad (1)$$

On the other side to satisfy the fixed flow RGE for $\tilde{\lambda}_1$ we need

$$0 = 2 \underbrace{\left(\tilde{\lambda}^2 - 4N_1 N_c \tilde{y}_1^4\right)}_{\text{should be } <0} + \underbrace{\tilde{\lambda}_1 \left(18\tilde{\lambda}_1 + 1 + 8N_1 N_c \tilde{y}_1^2\right)}_{>0}$$

But due to (1)

$$\begin{aligned} \left(\tilde{\lambda}^2 - 4N_1 N_c \tilde{y}_1^4\right) &= \left(\tilde{\lambda} - 2N_1 N_c \tilde{y}_1^2\right)\left(\tilde{\lambda} + 2N_1 N_c \tilde{y}_1^2\right) \\ &+ \underbrace{4N_1 N_c \tilde{y}_1^4 (N_1 N_c - 1)}_{\geq 0} > 0 \end{aligned}$$

$\rightarrow$ contradiction!

QED

We found that a UV complete theory with 2 scalar singlets and a $Z_2 \times Z_2$ symmetry is in a symmetric phase at high temperature

(opposite of the result for Weinberg's UV non-complete theory)

Easy to generalise to

- include also Majorana fields, not only Dirac (fermion number not conserved)
- take arbitrary representations of the fermions under the gauge group
- arbitrary (product of) gauge group(s)

with the same conclusion

Asymptotically safe theories

The other possibility for the UV limit of field theories is

$$p_i \xrightarrow{t\to\infty} constant \neq 0$$

Only if constants small these theories are perturbative in the UV.

Different from UV free theories (perturbative by definition there)

The first explicit example was given only relatively recently

Litim,Sannino '14

The situation more complicated than in the free case because higher loops needed

At 1-loop:

$$\frac{dg^2}{dt} = -b_0 g^4$$

In free case $b_0 > 0$ and $g^2 = 1/(2b_0 t)$

But we are interested in $b_0 < 0$. Obviously 2-loop is needed

$$\frac{dg^2}{dt} = |b_0| g^4 + C g^6$$

A non-trivial ($g \neq 0$) is obtained when the r.h.s. of the RGE vanishes:

$$|b_0|g^4 + Cg^6 = 0$$

This is similar to the Banks-Zaks fixed point, with $IR \leftrightarrow UV$

- to counter-balance the 1-loop term with the 2-loop term one needs an anomalously small (negative) 1-loop term, otherwise perturbation is lost

$$b_0 = \frac{11}{3}T(G) - \frac{2}{3}T(F) - \frac{1}{6}T(S)$$

T Dynkin index for Gauge bosons, Weyl Fermions and real Scalars

In the Veneziano limit $N_c, N_f \to \infty$ with fixed

$$0 < \epsilon = \frac{N_f}{N_c} - \frac{11}{2} \ll 1$$

one gets a small and negative

$$b_0 = -\epsilon$$

- one needs $C < 0$

  However computation gives $C > 0$.

  $\rightarrow$ Yukawa needed, RGE

$$\frac{dg^2}{dt} = g^2 \left(\epsilon g^2 + g^4 - y^2\right)$$

  RGE for Yukawa:

$$\frac{dy^2}{dt} = y^2 \left(y^2 - g^2\right)$$

  UV fixed point perturbative ($\epsilon \ll 1$)

$$g^2 \sim \epsilon \quad , \quad y^2 \sim \epsilon$$

- because of Yukawa we need scalars, $\rightarrow$ quartic couplings RGEs:

$$\frac{d\lambda}{dt} = \lambda^2 + \lambda y^2 + g^4 - \lambda g^2 - y^4 = 0 \rightarrow \lambda \sim \epsilon$$

  All this was very schematic

Situation harder than in asympotically free case

- in the free case everything is perturbative by definition for $t \to \infty$; here not any more, only if $\epsilon \ll 1$
- higher loops needed

Solutions for safe case are harder to classify

Fortunately for the previous case of two scalar singlets one does not need to solve anything.

The previous (free) equation

$$-\tilde{\lambda}_1 = 18\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 + 8N_1N_c\tilde{\lambda}_1\tilde{y}_1^2 - 8N_1N_c\tilde{y}_1^4 \qquad (2)$$

was coming from

$$\frac{d\lambda_1}{dt} = 18\lambda_1^2 + 2\lambda^2 + 8N_1N_c\lambda_1 y_1^2 - 8N_1N_c y_1^4$$

Now (safe) the eq. *at the free fixed point in the UV* is

$$0 = 18\lambda_1^2 + 2\lambda^2 + 8N_1N_c\lambda_1 y_1^2 - 8N_1N_c y_1^4$$

Same as (2) but without l.h.s. and no tilde

High T mass the same as before (but without tilde)

So without furher computation we conclude:

Any UV complete (either free or safe) theory with 2 scalar singlets and a $Z_2 \times Z_2$ symmetry is in a symmetric phase at high T

Conclusions

- Symmetry non-restoration is a very interesting phenomenon with several applications in theories with a UV cutoff (M_{Planck}?). For

$$m \ll T \ll M_{Planck} \to \text{ symmetry broken}$$

- Here we tried to check if the same can be obtained also for

$$M_{Planck} \to \infty$$

- although we cannot claim a no-go, all examples with no UV cutoff lead to symmetry restoration

- the reason is that

  1. in the usual finite M_{Planck} case the models with non-restoration had an intrinsic cutoff

  $$\Lambda_{Landau} \gg M_{Planck}$$

     so with $M_{Planck} \to \infty$ we cross Λ_{Landau}

  2. if we correct the models to avoid Λ_{Landau}, symmetry non-restoration disappears

- in a sense this is not that strange; we know already that symmetry non-restoration disappears when we restrict the parameter space: supersymmetric theories cannot have bosonic symmetry non-restoration (Higgs quartics are essentially Yukawas in susy $\lambda \sim y^2$)

Haber '82, Mangano '84

- other checks (cases with gauge non-singlet scalars, more than two scalars, etc) also point to symmetry restoration
- it would be interesting to prove a no-go similar to susy case or find a counter-example

Thank you !