Holography Abhors Visible Trapped Surfaces

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Outline

1. Introduction

2. Cosmic Censorship:
What, Why and Current Status

3. Tech Review:
Holography and Coarse Grained Spacetimes

4. Trapped Surfaces are Hidden by Event Horizons in AdS/CFT

Introduction

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- **Bad News:** Many violations of WCCC when studying general dimensions and asymptotics or fine-tuned initial data.
- **Good news!:** in classical holography, we proved that any trapped surface (and thus its implied singularity) is hidden by an event horizon.

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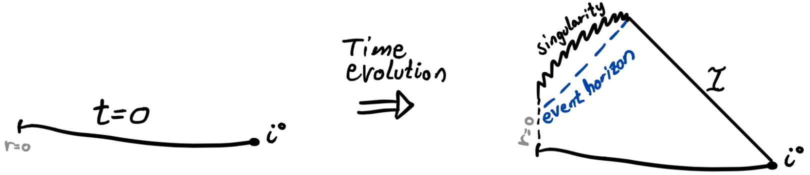
Cosmic Censorship:
What, Why and Current Status

The Statement of Cosmic Censorship

Weak cosmic censorship conjecture: *nonsingular initial data evolves to a spacetime with a complete asymptotic infinity* $\mathscr{I}$



Why Care

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  - Black holes are the typical endpoints of gravitational collapse, and GR provides fully deterministic evolution outside of them.
  - Enables proof of many results, such as:
    - Hawking area theorem
    - Black holes cannot bifurcate
    - Trapped surfaces are hidden behind horizons
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    - The Penrose Inequality*
- **If WCCC false in GR but true in classical limit of quantum gravity:**
  - The above list of results possible not true in all theories, but at least in the ones we care about.
  - Potential way to identify theories that cannot be UV-completed in quantum gravity.

The Status of Cosmic Censorship

WCCC is false in GR without additional demands:

Demand	
Null energy condition	(✗ Throw negative null energy into AdS)
Generic initial data	(✗ Fine tuned massless Klein-Gordon field)[1]
$D < 5$	(✗ Black strings in $D \geq 5$)[2]
Not asymptotically AdS_4	(✗ Inhomogenous charged black branes)[3]
Fundamental matter	(✗ Pressureless fluid)[4]

Validity of WCCC for generic initial data in asymptotically flat $D = 4$ remains a possibility, but AdS_4 violation suggests another option.

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1 [Choptuik '93], [Christodoulou '93], [Hamade, Stewart, 9506044]
2 [Gregory, LaFlamme 9301052, 9404071], [Lehner, Pretorius 1006.5960]
3 [HSW 1604.06465], [CS 1702.05490], [CHS 1709.07880, 1805.06469], [HS 1901.11096] by Crisford, Horowitz, Santos, and Way.
4 [Shapiro, Teukolsky '91]

WCCC as a statement about quantum gravity

WCCC-violating branes in AdS_4 are constructed in Einstein-Maxwell theory.

Including a charged scalar field of mass m and charge q removes the WCCC-violation precisely when

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This is exactly the statement of the **weak gravity conjecture** [Arkani-Hamed, Motl, Nicolis, Vafa 0601001]

WCCC as a statement about quantum gravity – Evidence

Question

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There now is evidence in favor of this in holography: two of the most important consequences of WCCC now derived directly in AdS/CFT

- The Penrose Inequality [Engelhardt, Horowitz 1903.00555]

$$M_{\mathrm{AdM}} \geq F\left(\mathrm{Area}[\tau]\right)$$

- Trapped surfaces lie behind horizons

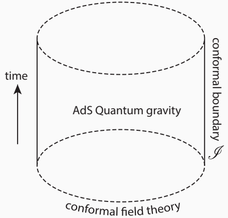
Tech Review:
Holography and Coarse Grained Spacetimes

AdS/CFT in one slide

Quantum gravity in an asymptotically Anti-de-Sitter spacetime

$$\Updownarrow$$

Quantum field theory at $\mathscr{I}$ with conformal symmetry



In the limit $G_N \to 0$ we get a duality between classical gravity and $N \to \infty$ CFT.

Entanglement entropy in AdS/CFT

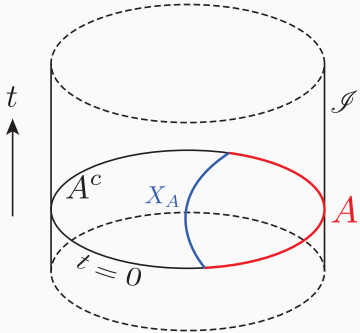
The RT/HRT formula gives a way to calculate CFT von-Neumann entropies through a gravity calculation:

$$S_{\mathrm{vN}}[\rho_A] = \frac{\mathrm{Area}[X_A]}{4 G_N \hbar}$$

where

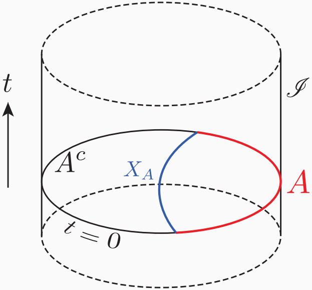
- A – spatial subregion of CFT
- $\rho_A = \mathrm{Tr}_{A^c} \left|\psi\right\rangle \left\langle\psi\right|$
- X_A – the smallest extremal surface with $\partial X_A = \partial A$ that is also homologous to A

Homologous: there exists a hypersurface Σ so that $\partial\Sigma = A \cup X_B$



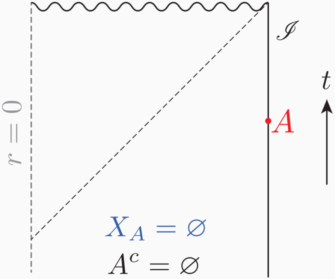
HRT surfaces - Examples

Example 1



$$S_{\text{vN}}[\rho_A] = \frac{\text{Area}[X_A]}{4G\hbar} = \text{UV-divergent}$$

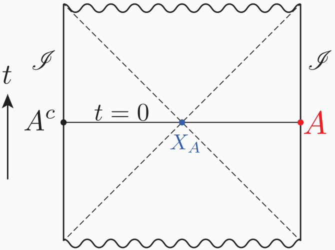
Example 2

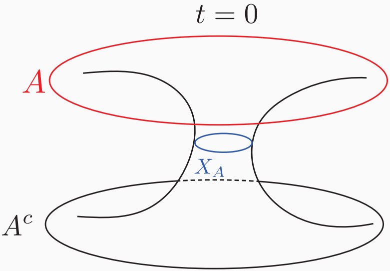


$$S_{\text{vN}}[\rho_A] = S_{\text{vN}}[|\psi\rangle\langle\psi|] = \frac{\text{Area}[X_A]}{4G\hbar} = 0$$

HRT surfaces - Examples

Example 3





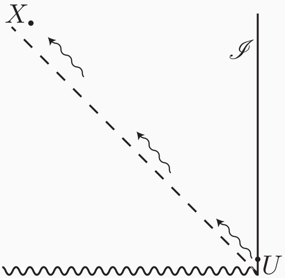
$$S_{\mathrm{vN}}[\rho_A] = \frac{\mathrm{Area}[X_A]}{4G_N\hbar} = \frac{\mathrm{Area}[\mathrm{horizon}]}{4G_N\hbar}$$

A crucial observation

- Acting with a CFT unitary with support only on A can never change $S_{\rm vN}[\rho_A]$:

$$S_{\rm vN}[\rho_A] = S_{\rm vN}[U\rho_A U^\dagger], \qquad \text{if } U = V_A \otimes 1_{A^c}$$

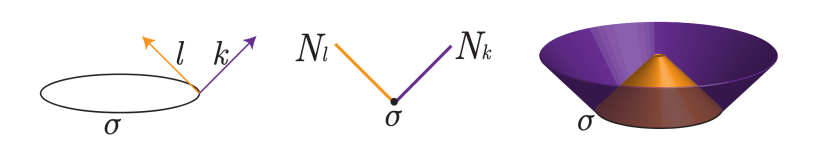
- **In the bulk, this means that X_A can never be timelike separated to A** [Wall 1211.3494], [Headrick et al. 1408.6300]
- Taking A to be a full connected component of $\mathscr{I}$, it means X_B must be spacelike to $\mathscr{I}$



Surfaces

surface σ = codimension 2 spatial submanifold

- Every surface has two null normal vector fields: k^a and ℓ^a
- Firing null geodesics along these normals gives *null congruences* N_k, N_ℓ



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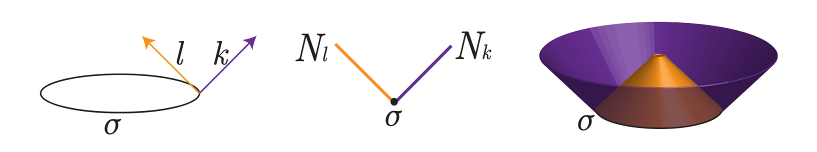


- The expansions θ_k and θ_ℓ measures the change in the area-element on the light-front

$$\theta_k = k^a \nabla_a \ln \sqrt{\det h_{bc}}, \qquad \theta_\ell = \ell^a \nabla_a \ln \sqrt{\det h_{bc}}$$

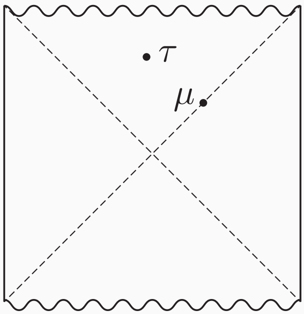
Trapped surfaces and apparent horizons

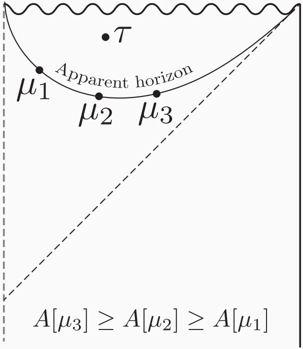
Trapped surface τ

$\theta_k < 0$, $\theta_\ell < 0$

Marginally trapped surface μ

$\theta_k = 0$, $\theta_\ell < 0$

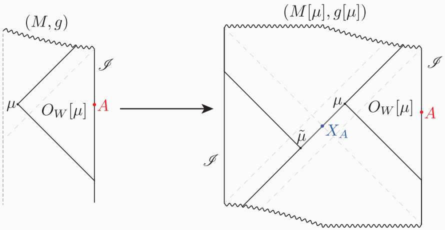




Coarse grained spacetimes

Provided μ is **minimar**, meaning it satisfies some additional technicalities, we can construct an associated **coarse grained spacetime**

[Engelhardt, Wall 1806.01281]



$S[\rho_A] = 0$

$$S[{\rho_A}^{\text{coarse}}] = \frac{\text{Area}[X_A]}{4G_N\hbar} = \frac{\text{Area}[\mu]}{4G_N\hbar} = \begin{pmatrix}\text{uncertainty about} \\ \text{what is behind } \mu\end{pmatrix}$$

Main observation

If μ was in causal contact with $\mathscr{I}$ in $M[\mu]$, then so would X_A, allowing us to change $S_{vN}[{\rho_A}^{\text{coarse}}]$ with a unitary on A! **Thus μ is behind the horizon in the coarse grained spacetime.**

Rest of the talk

Rest of the talk: μ must also be behind the horizon in the original spacetime. Furthermore, the same applies for generic trapped surfaces τ.

Trapped Surfaces are Hidden by Event Horizons in AdS/CFT

The theorem

Theorem

If there exists a past well-behaved trapped surface τ *in a classical asymptotically AdS spacetime* (M, g) *satisfying the null energy condition, then at least one of the following holds:*

- (M, g) *has an event horizon and* τ *lies behind it*
- (M, g) *has no holographic dual*
- *Classical GR admits NEC-preserving solutions with evaporating singularities*

We also assume no closed timelike curves and a genericity condition which we believe can be proved directly.

We **cannot** assume global hyperbolicity or strong asymptotic predicatbility

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Assuming classical GR has no NEC-preserving evaporating singularities, we then have in holographic spacetimes that

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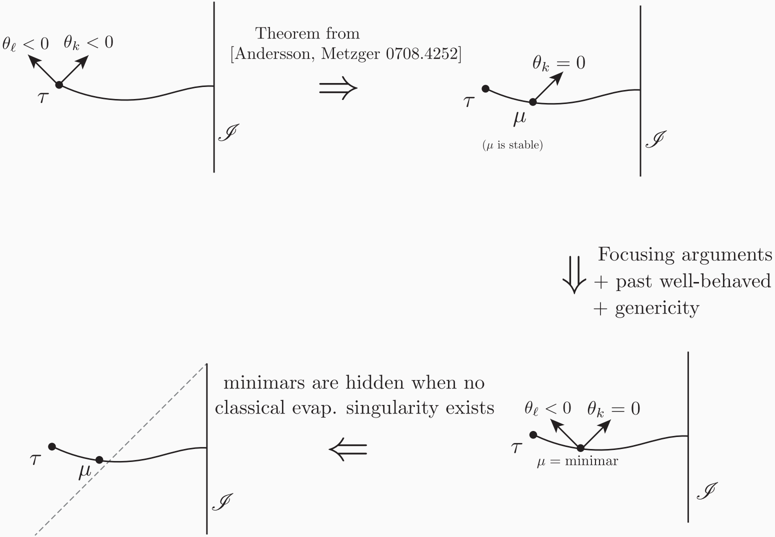
- If no holographic dual means no UV-completion in QG, then naked trapped surfaces potential swampland condition.
- In the quantum theory, if a trapped surface is outside the horizon, the any signal from it should not reach $\mathscr{I}$ before a time of order $\mathcal{O}(t_{\text{page}})$

Structure of proof

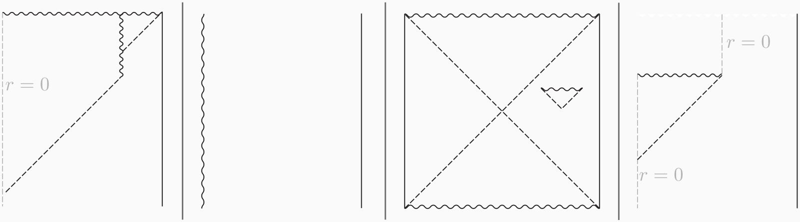


Evaporating singularity

Definition

An asymptotically AdS spacetime (M, g) is said to be devoid of evaporating singularities if

- If an event horizon exists, it reaches i^+
- In the conformal completion, every set bounded by a closed compact hypersurface has a compact interior.



For the argument that this assumption suffices to show that μ is behind the horizon in both $M[\mu]$ and M, see the paper.

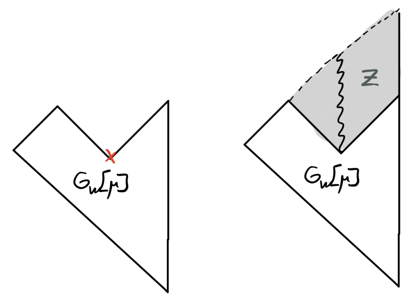
Thank you for listening!

μ behind the horizon in M

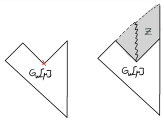
Let Z be an extension of $O_W[\mu]$ past potential Cauchy horizons such that

1. Z is spacelike to μ in $O_W[\mu] \cup Z$
2. Z is maximal, subject to the above.

The original spacetime induces a particular choice of Z. Let us choose this.



μ behind the horizon in M



If $i^+ \subset Z$ we are done, since i^+ is spacelike to μ, and by the absence of closed timelike curves, this must remain true in the full spacetime M that contains $O_W[\mu] \cup Z$.

If $i^+ \not\subset Z$, then CFT evolution proceeds further. This means that Z can be extended even further,[5] but since Z was a maximal extension subject to (1), any further extension[6] must put μ into causal contact with $\mathscr{I}$, which yields a pathological coarse grained spacetime. Thus $i^+ \not\subset Z$ contradicts the HRT proposal.

[5] If it cannot, then spacetime breaks down, and μ lies behind the horizon in the sense that it can send no signals to $\mathscr{I}$ through spacetime.

[6] Including any one compatible with data on $N_{-k}[\mu]$