Hydrodynamic Fluctuations in Spin Chains and QFTs

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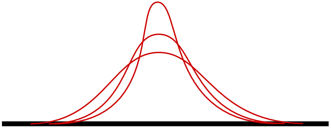
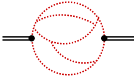
weak fluctuations



Controlled EFT

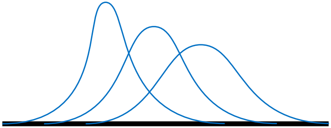
- Slow thermalization of $SU(2)$ spin chains arXiv:2007.13753 with Glorioso Chen Nandkishore Lucas

- 'Diffuson cascade'

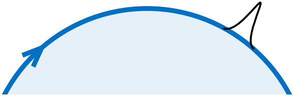
$G(t,k) \sim$  $\sim e^{-\sqrt{Dk^2 t}}$

arXiv:2006.01139

strong fluctuations



KPZ: $\omega \sim ck - i\mathcal{D}k^{3/2}$

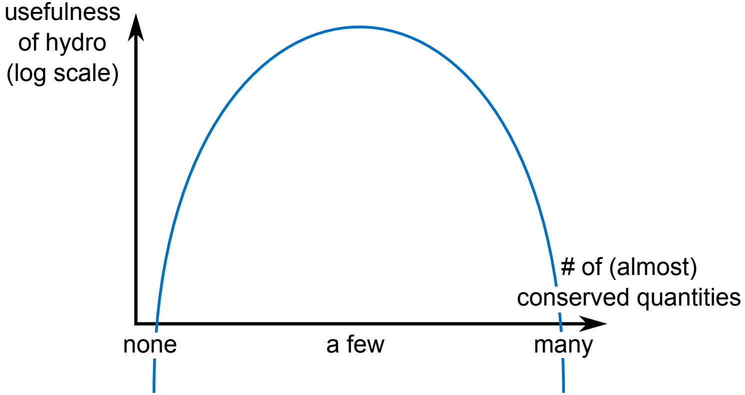


- Edge modes in QH systems
  PRL **124** (2020) with Paolo Glorioso

- 2d QFTs
  w.i.p. using Hamiltonian truncation with Fitzpatrick Katz Walters

Universality of Hydrodynamics

Any thermalizing system, quantum or classical, is described by hydrodynamics at sufficiently late times



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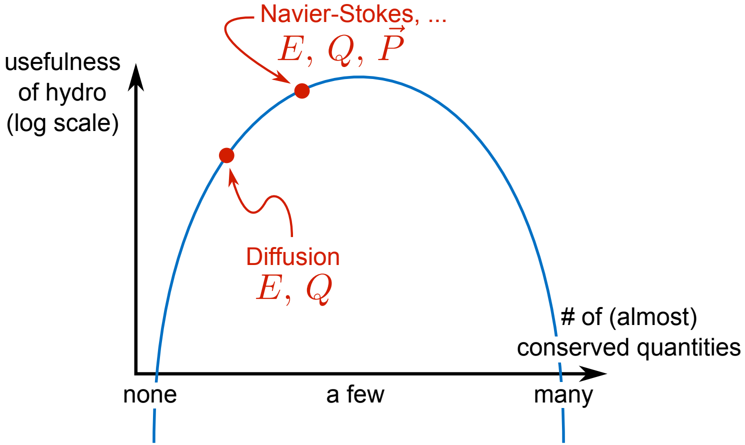

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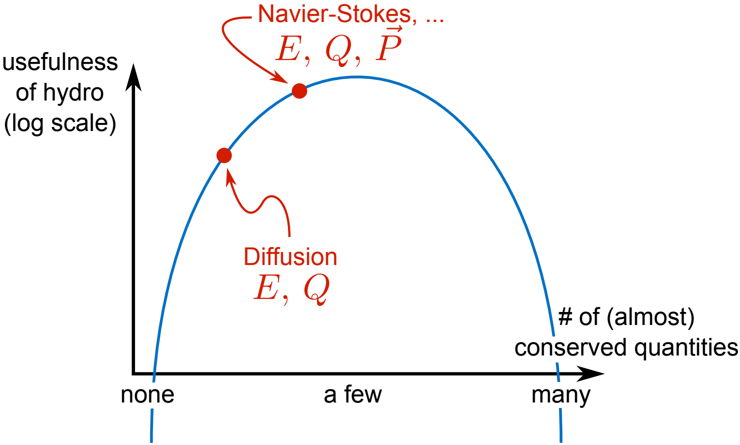
How is such a universality possible?



Universality of Hydrodynamics

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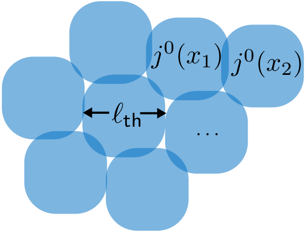
How is such a universality possible?

Most excitations 'relax' at finite $T \rightsquigarrow$ thermalization time $\tau_{\rm th}$

Conserved densities j^0 related to symmetries decay with rate $\Gamma \sim k^2$

For k small enough these are parametrically slower than generic excitations

Hydrodynamics is the late time ($t \gg \tau_{\rm th}$) description of these 'coarse grained' quantities



How it works

Theory of a conserved density $n = j^0$ (or its potential μ), subject to

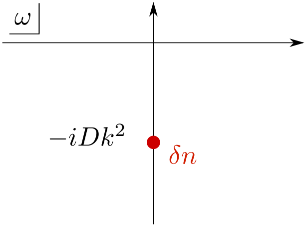
$$\dot{n} + \nabla \cdot j = 0$$

This also involves j^i. Close the equation with a constitutive relation

$$j_i = -D\partial_i n + \cdots$$

Solving these equations yields a diffusive Greens function

$$G^R_{nn}(\omega, k) = \frac{\chi D k^2}{-i\omega + Dk^2} + \cdots$$



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Two expansions: gradients $\partial + \partial^2 + \cdots$ and fluctuations $\delta n + \delta n^2 + \cdots$

Always controlled
(in principle)

Controlled when interactions
are *irrelevant*

How it works II

Theory of conserved densities ϵ, π_i or their potentials $T(x)$, $v_i(x)$, subject to $\dot{\epsilon} + \nabla \cdot j^\epsilon = 0\,, \quad \dot{\pi}_i + \partial_j \tau_{ij} = 0$.

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We again close the equations with constitutive relations

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This also involves the currents j^{ϵ}, τ_{ij}

We again close the equations with constitutive relations

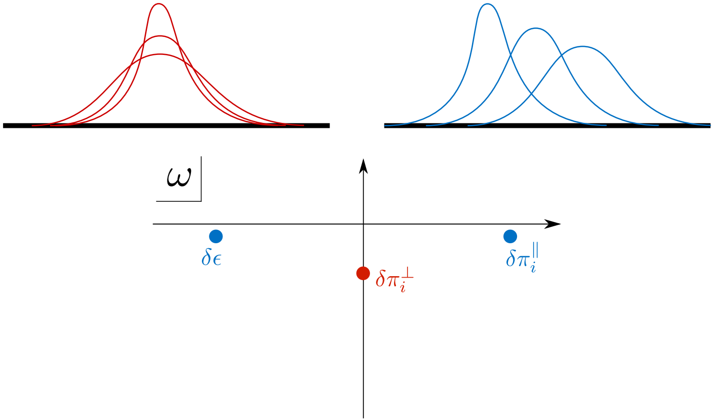
Solving around equilibrium $v_i(x) = 0 + \delta v_i$ and $T(x) = T + \delta T$ gives

$$G^R_{\pi_i \pi_j}(\omega, k) \simeq \frac{k_i k_j}{k^2} \underbrace{\frac{\omega^2}{c_s^2 k^2 - \omega^2 - i\Gamma k^2 \omega}}_{\text{sound}} + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \underbrace{\frac{Dk^2}{-i\omega + Dk^2}}_{\text{diffusion}}$$

Ballistic v. Diffusive

$$\omega = -iDk^2 + \cdots$$

$$\omega = \pm ck - \frac{i}{2}\Gamma k^2 + \cdots$$



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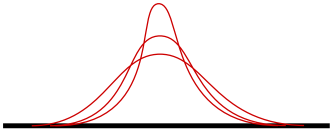

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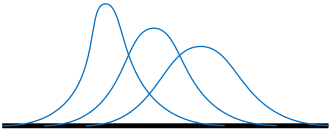

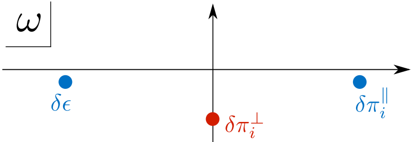

- heat
- particle number
- spin
- $\cdots$

- regular sound
- superfluid sound
- spin waves
- transverse sound
- $\cdots$

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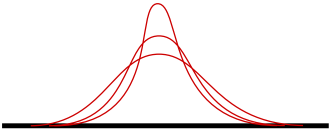

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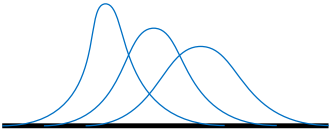

Other possibilities, e.g.:

- $\omega = ck \sin\theta - iDk^2$ (smectic, magnetohydrodynamics)
- $\omega = \pm k^2 - ik^2$ (nematic)
- $\omega = \pm k^2 - ik^4$ (spin waves in a ferromagnet)
- $\cdots$

Ballistic v. Diffusive

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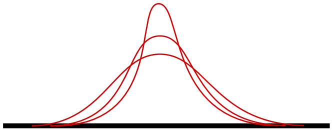

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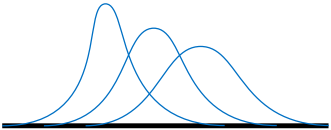

When are fluctuations big? Diffusive:

$$0 = \dot{n} - \nabla[D\nabla n] + \cdots$$

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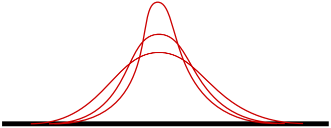

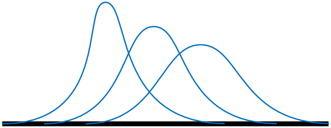

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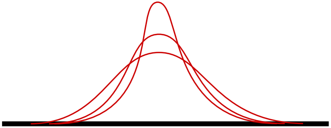

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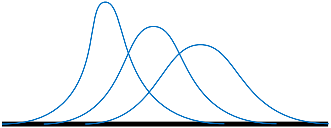

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$$0 = \dot{n} - \nabla[D(n)\nabla n] + \cdots$$

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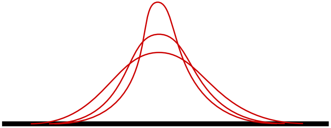

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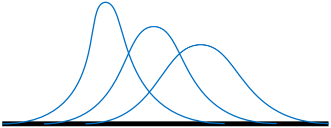

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We need to know how charge fluctuations scale: scaling $\omega \sim k^2$

$$\langle n(x,t)n\rangle \propto \frac{e^{-x^2/4Dt}}{t^{d/2}} \quad \Rightarrow \quad \delta n \sim \omega^{d/4} \sim k^{d/2}$$

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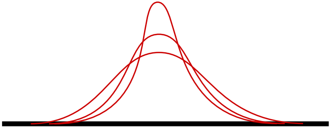

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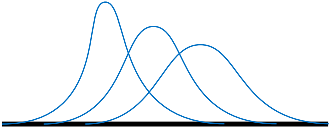

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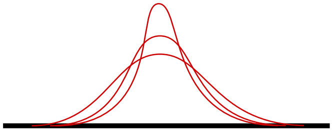

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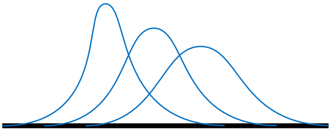

When are fluctuations big? Diffusive: when $d \leq d_c = 0$

$$\begin{aligned} 0 &= \dot{n} - \nabla[D(n)\nabla n] + \cdots \\ &= \left(\partial_t - D\nabla^2 - D'\delta n \nabla^2 + \cdots\right)\delta n \end{aligned}$$

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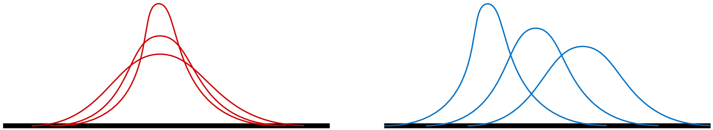

When are fluctuations big? Ballistic:

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In the reference frame of the pulse $x' = x - ct$ we again scale $\omega' \sim k^2$

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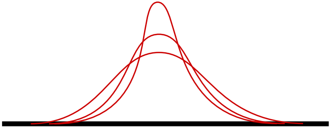

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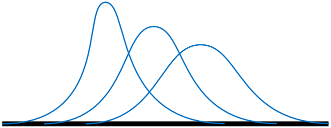

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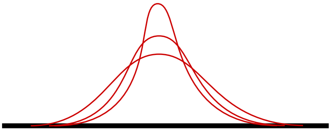

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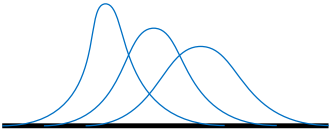

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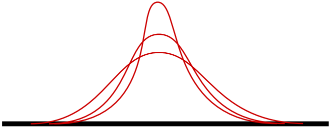

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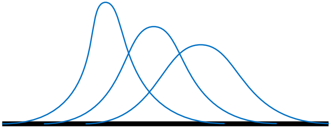

When are fluctuations big? Ballistic: when $d \leq d_c = 2$

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$$= \big(\partial_{t'} \qquad + c'\delta n\nabla - D\nabla^2 + \cdots\big)\,\delta n$$

$$\sim k^2 \qquad \sim k^{1+\frac{d}{2}} \qquad \sim k^2$$

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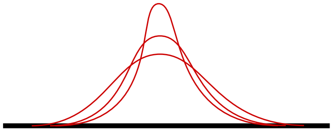

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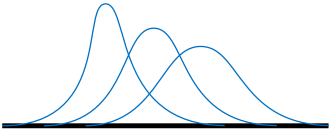

Bottomline:

	Diffusive modes	Ballistic modes
Weak fluctuations	$d > 0$	$d > 2$
Strong fluctuations	never	$d < 2$

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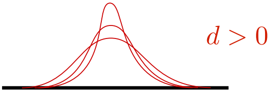
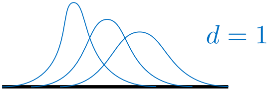
2. Weak fluctuations
3. Strong fluctuations





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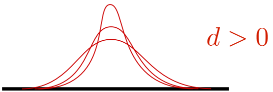
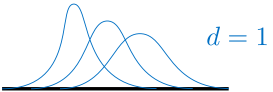
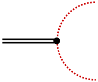




Irrelevant interactions

We found that diffusive fluctuations $\delta n \sim k^{d/2}$ are irrelevant in $d > 0$

$$j_i = D\partial_i n + D' n \partial_i n + \cdots$$

$=$  $+$  $+ \cdots$

Studied within theory of hydrodynamic fluctuations

Martin Siggia Rose '73, Forster Nelson Stephen '77

(long history: Zwanzig '61 Mori '65 Kawasaki '68 Alder Wainwright '70 Ernst Hauge van Leeuwen '70 Pomeau Résibois '75 ...)

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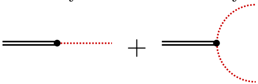
=  + ⋯

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$$j_i = D\partial_i n + D' n \partial_i n + \xi_i + \cdots \ , \qquad \langle \xi_i(x,t)\xi_j\rangle = 2\chi D T \delta^d(x)\delta(t)\delta_{ij}$$

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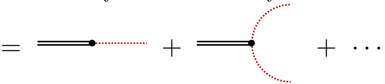

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Martin Siggia Rose '73, Forster Nelson Stephen '77

(long history: Zwanzig '61 Mori '65 Kawasaki '68 Alder Wainwright '70 Ernst Hauge van Leeuwen '70 Pomeau Résibois '75 ...)

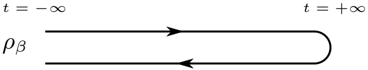
$$j_i = D\partial_i n + D' n\partial_i n + \xi_i + \cdots \ , \qquad \langle \xi_i(x,t)\xi_j\rangle = 2\chi D T \delta^d(x)\delta(t)\delta_{ij}$$

Modern approach: path integral on a Schwinger-Keldysh contour

Kamenev '11, Grozdanov Polonyi '13, Crossley Glorioso Liu '15, Haehl Loganayagam Rangamani '15, Jensen Pinzani-Fokeeva Yarom '17

Roughly: $n \sim \phi_{\text{top}} + \phi_{\text{bottom}}$

$\xi \sim \phi_{\text{top}} - \phi_{\text{bottom}}$



Irrelevant interactions

We found that diffusive fluctuations $\delta n \sim k^{d/2}$ are irrelevant in $d > 0$

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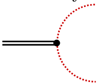
$=$  $+$  $+ \cdots$

For this talk, a simplified treatment of hydro fluctuations will be enough to illustrate concepts Ernst Hauge van Leeuwen '70, Kovtun Yaffe '03

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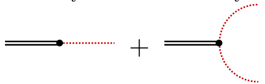
$= $  $+ \cdots$

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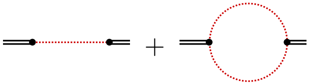
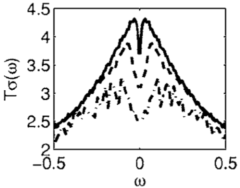
$$\sigma(\omega) = \frac{1}{2T}\langle jj\rangle(\omega, k=0) = \chi D + \#|\omega|^{d/2} + \#\omega + \cdots$$

$= $  $+ \cdots$



Mukerjee Oganesyan Huse '05

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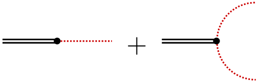
$=$  $+ \cdots$

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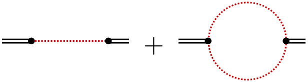
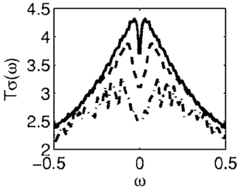
$=$  $+ \cdots$



Mukerjee Oganesyan Huse '05

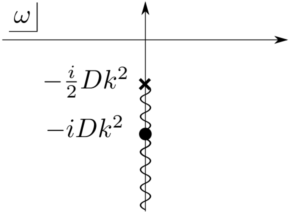
Long-time tails

- Discovered in molecular dynamics numerics Alder Wainwright '70
- Seen in AdS/CFT Caron-Huot Saremi '09
- Recent interest for RHIC

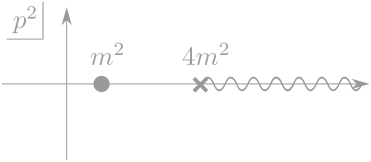
Analytic structure

These calculations can also be performed at finite k Chen-Lin LVD Hartnoll '18

$$G^R_{nn}(\omega,k) = \frac{\chi D k^2 + \cdots}{-i\omega + Dk^2 + \Sigma k^2}\,, \qquad \Sigma(\omega,k) = (\#i\omega + \#k^2)\left[k^2 - \frac{2i\omega}{D}\right]^{\frac{d-2}{2}}$$



Relativistic massive particle $G(p^2)$



ANALYTIC STRUCTURE

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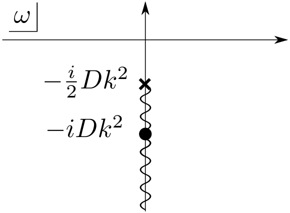

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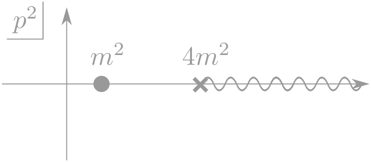
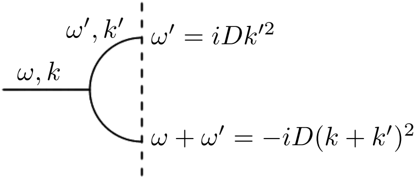


Two-'diffuson' threshold:



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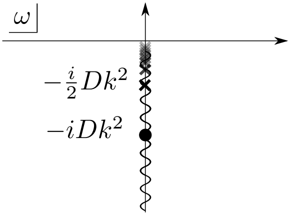

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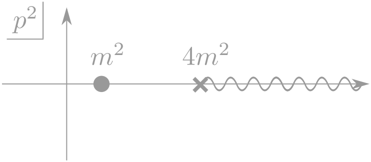

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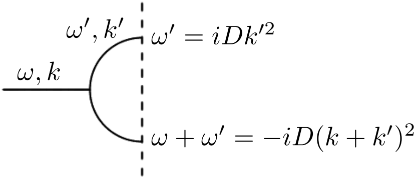

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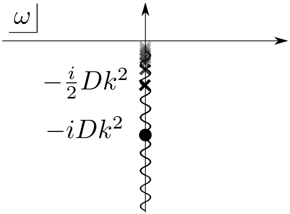

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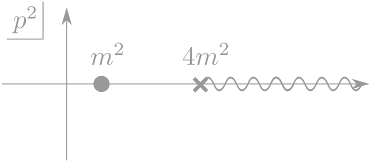
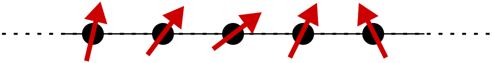


$$G_{nn}(t, x=0) = \frac{1}{(Dt)^{d/2}}\left(1 + \frac{\#}{t^{d/2}} + \cdots\right)$$

Thermalization in spin chains



$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

Thermalization and hydrodynamics of this model long debated

Srivastava Liu Viswanath Müller '94, ... , Bagchi '13, Das Chakrabarty Dhar Kundu Huse Moessner Ray Bhattacharjee '18, Gamayun Miao Ilievski '19

Recently revived by De Nardis Medenjak Karrasch Ilievski '20

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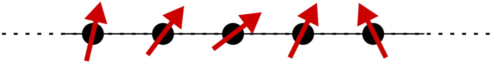

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Hydrodynamic description:

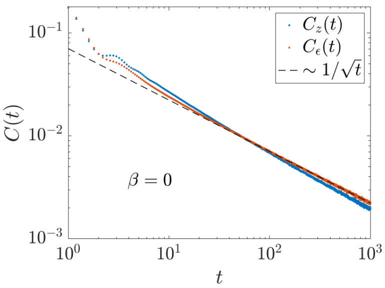
Symmetry	Conserved density	
$SU(2)$	$n^a(x)$	'coarse grained' S_i^a, $a = x, y, z$
time translation	$\epsilon(x)$	'coarse grained' $J\vec{S}_i \cdot \vec{S}_{i+1}$

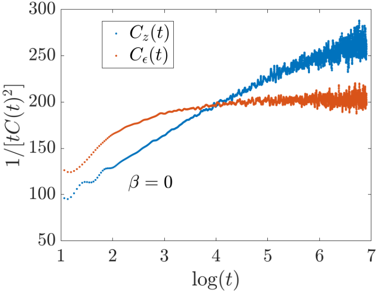
To leading order, all densities diffuse

$$\langle n^a(x,t) n^b \rangle = \delta^{ab} \frac{e^{-x^2/4D|t|}}{|t|^{1/2}} \quad \Rightarrow \quad \langle S_i^z(t) S_i^z \rangle \sim \frac{1}{|t|^{1/2}}$$

Thermalization in spin chains

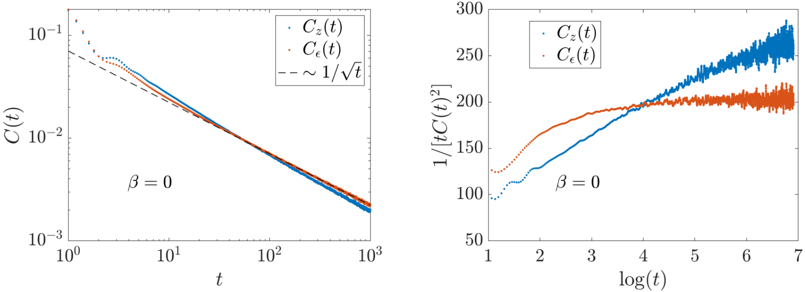
De Nardis Medenjak Karrasch Ilievski '20





Slow thermalization... subdiffusion? $C(t) \sim 1/(t \log^\alpha t)^{1/2}$

De Nardis Medenjak Karrasch Ilievski '20



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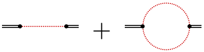
Hydrodynamics really predicts

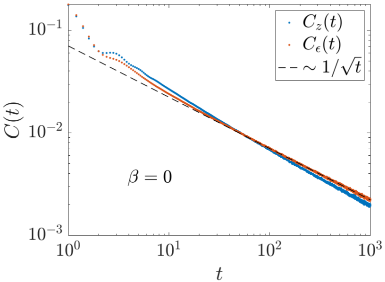
$C(t) =$  $+ \cdots$

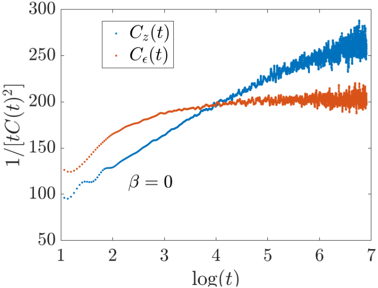
$$= \frac{1}{\sqrt{t}} \left(1 + \frac{a}{\sqrt{t}} + \cdots \right)$$

with $a \sim \sqrt{\tau_{\mathrm{th}}}$

Thermalization in spin chains

De Nardis Medenjak Karrasch Ilievski '20





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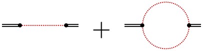
$C(t) =$  $+ \cdots$

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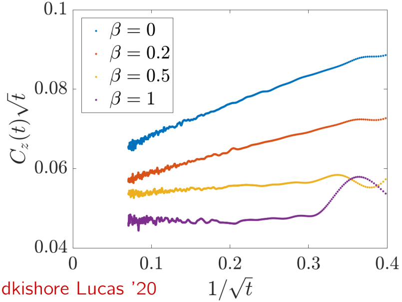
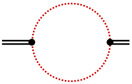


Glorioso LVD Chen Nandkishore Lucas '20

Diffuson cascade

$\langle \mathcal{O}\mathcal{O} \rangle (t,k) =$  $+$  $+ \cdots$

$$\sim \quad g(t,k) \quad + \quad k^d g(t, \tfrac{k}{2})^2 \quad + \quad \cdots$$

$$\sim \quad e^{-Dk^2 t} \quad + \quad k^d e^{-Dk^2 t/2} \quad + \quad \cdots$$

For $\tau_{\mathrm{th}} \lesssim t \lesssim \dfrac{1}{Dk^2}$, the first term dominates

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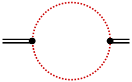
$\langle \mathcal{O}\mathcal{O} \rangle (t,k) =$  $+$  $+ \cdots$

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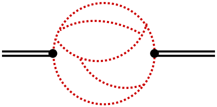
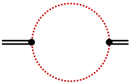
The n-diffuson contributions take the form

 $\sim \quad n!\,(k\ell_{\rm th})^{dn}\, e^{-Dk^2 t/n}$

At late times $\dfrac{1}{Dk^2} \lesssim t$, the term that dominates have $n(t) \simeq \sqrt{\dfrac{Dk^2 t}{d \log \frac{1}{k\ell_{\rm th}}}}$

Plugging back gives $\langle \mathcal{O}\mathcal{O} \rangle (t,k) \sim e^{-\sqrt{Dk^2 t}}$!

Diffuson cascade

$\langle \mathcal{O}\mathcal{O}\rangle(t,k) =$  $+$  $+ \cdots$

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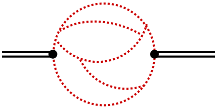 $\sim \quad n!\,(k\ell_{\rm th})^{dn}\, e^{-Dk^2t/n}$

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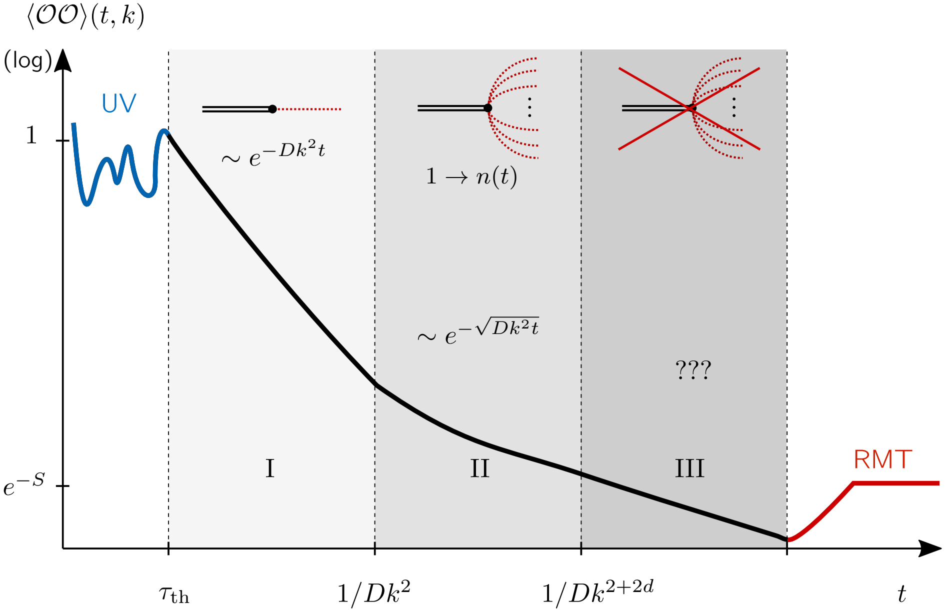
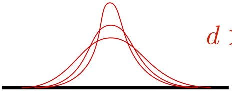
Plugging back gives $\langle \mathcal{O}\mathcal{O}\rangle(t,k) \sim e^{-\sqrt{Dk^2t}}$! LVD arXiv:2006.01139

At even later times $\frac{1}{k^{2+2d}} \lesssim t$: breakdown of hydro! and w.i.p. with Xiao Chen

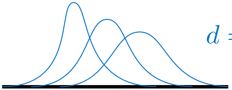
A RICHER STORY AT FINITE k



Contents



$d > 0$

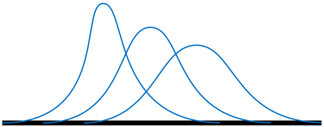


$d = 1$

Strong hydrodynamic fluctuations

We found that hydrodynamic fluctuations were large (relevant) for ballistic modes in 1+1d

$$\omega = ck - \frac{i}{2}\Gamma k^2 + \cdots$$



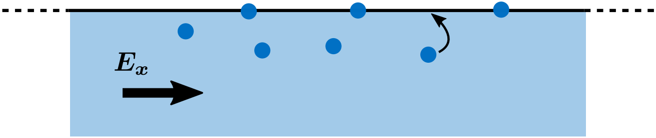
We will consider two situations where this occurs:

- Diffusion of $U(1)$ charge with a chiral anomaly
- Translation invariance: 2d QFTs

The anomaly

Single $U(1)$ conserved charge like before, but with an anomaly

$$\partial_\mu j^\mu = \nu \epsilon^{\alpha\beta} F_{\alpha\beta} \qquad \text{or} \qquad \dot{n} + \partial_x j_x = \nu E_x$$



Account for anomalies in constitutive relations Son Surowka '09

$$j_x = \nu\mu - \chi D \partial_x \mu + \cdots$$

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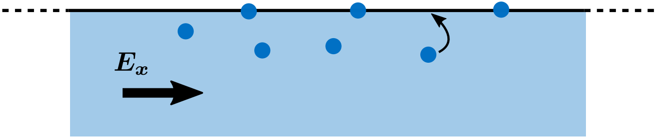

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$$j_x = \nu\mu - \chi D \partial_x \mu + \cdots$$

'Anomalous diffusion' equation:

$$0 = \dot{n} + c\partial_x n - \partial_x (D \partial_x n) + \cdots \qquad \text{with} \quad c = \nu/\chi$$

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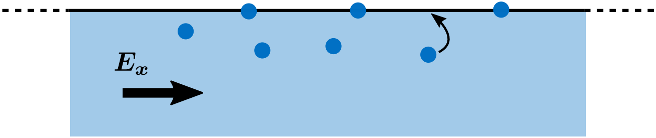

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Solving again for the Green's function gives Kane Fisher '95

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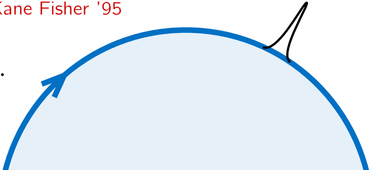

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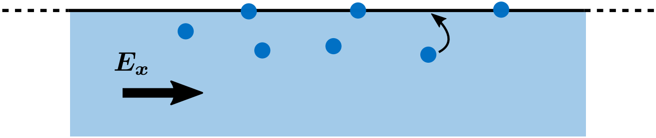

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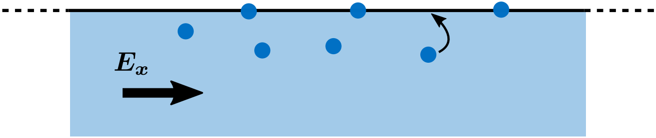

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$$j_x = \nu\mu - \chi D \partial_x \mu + \cdots$$

'Anomalous diffusion' equation:

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Solving again for the Green's function gives Kane Fisher '95

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Single $U(1)$ conserved charge like before, but with an anomaly

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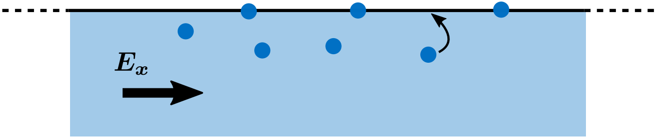

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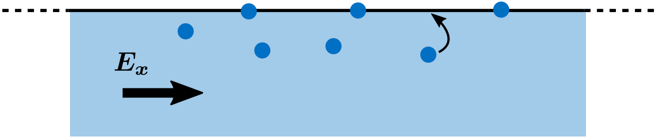

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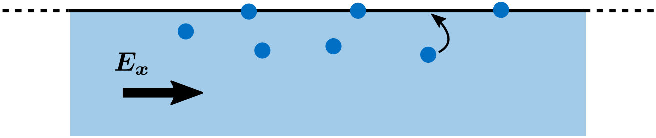

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'Anomalous diffusion' equation: (recall $\delta n \sim k^{d/2} = k^{1/2}$)

$$0 = \dot{n} + c\partial_x n - \partial_x(D\partial_x n) + c' n \partial_x n + \cdots$$

$$c\partial_x n \sim k^{3/2}, \quad \partial_x(D\partial_x n) \sim k^{5/2}, \quad c' n \partial_x n \sim k^2$$

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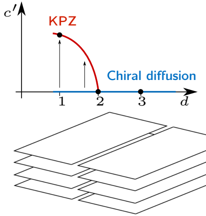
$\rightsquigarrow$ breakdown of diffusion!

What to do?

- Dim reg: expand from upper critical dimension $d_c = 2$.

  The theory at $d_c = 2$ describes chiral surface metals

- Exact solution for $d = 1$?



Burger's equation Forster Nelson Stephen '77, KPZ Kardar Parisi Zhang '86, 1d Navier-Stokes Narayan Ramaswamy '02

KPZ universality on the edge

'Anomalous diffusion' equation:

$$0 \quad = \quad \dot{n} \quad + \quad c\partial_x n \quad - \quad \partial_x(D\partial_x n) \quad + \quad c' n\partial_x n \quad + \quad \cdots$$

Follow chiral front: $x' = x - ct$, so $\partial_{t'} = \partial_t + c\partial_x$

$$0 \quad = \quad \partial_{t'} n \quad - \quad \partial_x(D\partial_x n) \quad + \quad c' n\partial_x n \quad + \quad \cdots$$

Map to KPZ equation $n \leftrightarrow \partial_x h$ Kardar Parisi Zhang '86

$$0 \quad = \quad \partial_{t'} h \quad - \quad D\partial_x^2 h \quad + \quad c'(\partial_x h)^2 \quad + \quad \cdots$$

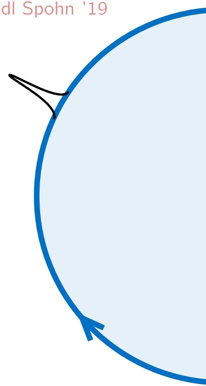
(the noise term also maps appropriately, as it must by fluctuation-dissipation)

Edge is in Burger's-KPZ universality, with $z = 3/2$!

Collective mode disperses as

$$\omega = ck - i\mathcal{D}k^z + \cdots \qquad \text{with} \quad \mathcal{D} = \sqrt{\frac{T}{\chi^3}\frac{|\nu|}{2\pi}|\chi'|} \quad \text{and} \quad z = \frac{3}{2}$$

similar dispersion relations observed in 1d hydro Narayan Ramaswamy '02 Spohn '14
but these are not robust vs disorder Das Damle Dhar Huse Kulkarni Mendl Spohn '19



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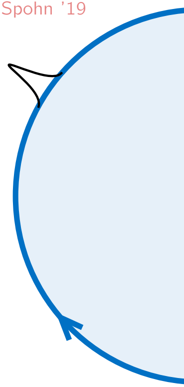

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Transport:

KPZ scaling function controls transport

$$G_{nn}(\omega, k) = \frac{\chi T}{\omega} g_{\mathrm{KPZ}}\left(\frac{\omega - ck}{\mathcal{D}k^z}\right) + \cdots$$

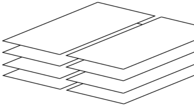
and gives

$$\sigma(\omega) = \lim_{k\to 0}\frac{\omega}{k^2}\,\mathrm{Im}\,G^R_{nn}(\omega, k) = \#\frac{\chi\mathcal{D}^{4/3}}{\omega^{1/3}} + \cdots$$

(g_{KPZ} known to high precision, with $\# \simeq 0.417816..$ Prähofer and Spohn '04)

Experiments

Singular edge transport: $$\sigma(\omega) \sim \frac{1}{\omega^{1/3}}$$



Anomalous damping of edge modes:

$$\omega \simeq ck - i\mathcal{D}k^{3/2} \qquad \text{with} \qquad \mathcal{D} = \sqrt{\frac{\chi'^2 T}{\chi^3}\frac{|\nu|}{2\pi}}$$

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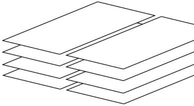

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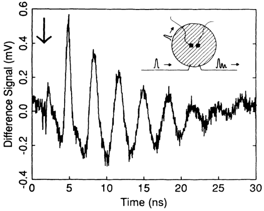
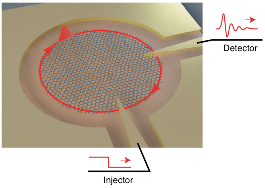


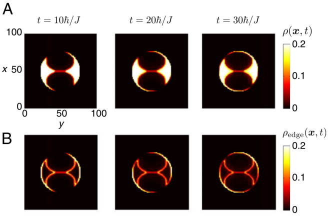
Ashoori Stormer Pfeiffer Baldwin West '92

GaAs



Kumada Glattli et al '14

Graphene



Goldman Spielman et al '13

Cold atoms

Experimental investigation of the damping of low-frequency edge magnetoplasmons in GaAs-$Al_xGa_{1-x}As$ heterostructures

V. I. Talyanskii,* M. Y. Simmons, J. E. F. Frost, M. Pepper, D. A. Ritchie, A. C. Churchill, and G. A. C. Jones
Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 OHE, United Kingdom
(Received 17 February 1994)

A detailed experimental study of damping and velocity of low-frequency edge magnetoplasmons in GaAs-$Al_xGa_{1-x}As$ heterostructures is presented. The damping is observed to be frequency dependent at filling factors close to integer values. The magnitude of the damping increases with frequency, the dependence being somewhere between linear and quadratic. This finding indicates that the damping of low-frequency edge magnetoplasmons cannot be described by the effective relaxation time. The experimental results are discussed in terms of existing models of low-frequency edge magnetoplasmons.

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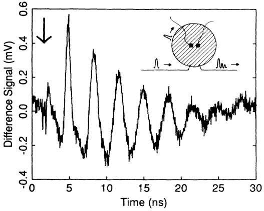

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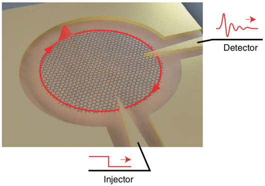

Kumada Glattli et al '14

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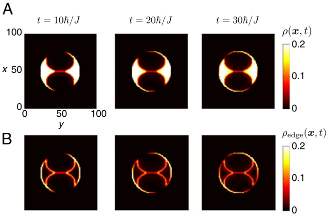

Goldman Spielman et al '13

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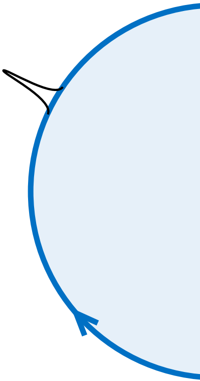
Neutral heat mode is similar, except if $\kappa_{xy} = 0$, like for $\nu = 2/3$

$$\omega \sim -i\mathcal{D}k^{5/3}$$

'Upstream' heat transport



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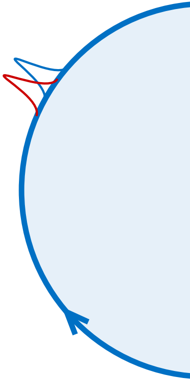

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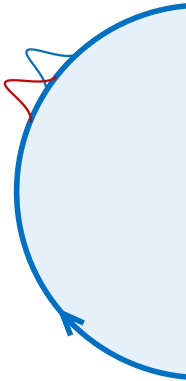

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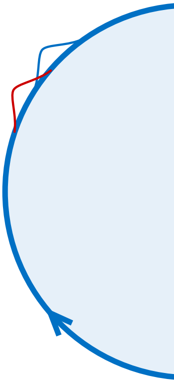

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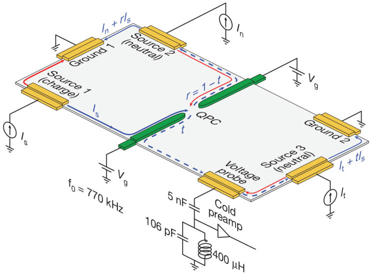

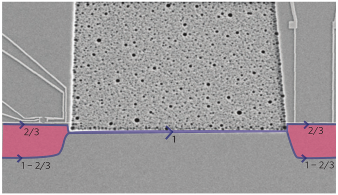

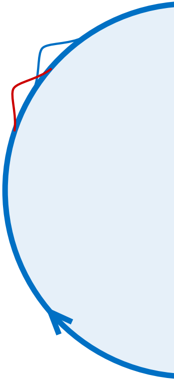

Bid Mahalu et al '**10**

Venkatachalam Hart Yacoby et al '**12**

Hydro in 2d QFTs

Translation invariance naturally leads to a ballistic mode

2d thermalizing QFTs will have KPZ dissipation

Can they be studied numerically without breaking translations?

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$\rightsquigarrow$ Lightcone Conformal Truncation

w.i.p. with Fitzpatrick Katz Walters

$$S = \int d^2x \, \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

Hamiltonian Truncation in a Nutshell

$$S = \int d^2x \, \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

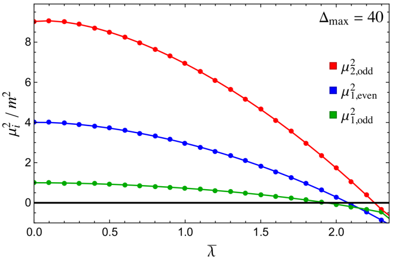
The UV is a free scalar theory

The two relevant deformations trigger a flow to the IR

Strategy:

- Build free theory Fock space $|n\rangle$ and truncate it $n_{\max}$
- Evaluate $\langle n|H_{\text{int}}|n'\rangle$ and diagonalize it

Pedagogical introduction: Anand Fitzpatrick Katz Khandker Walters Xin 2005.13544



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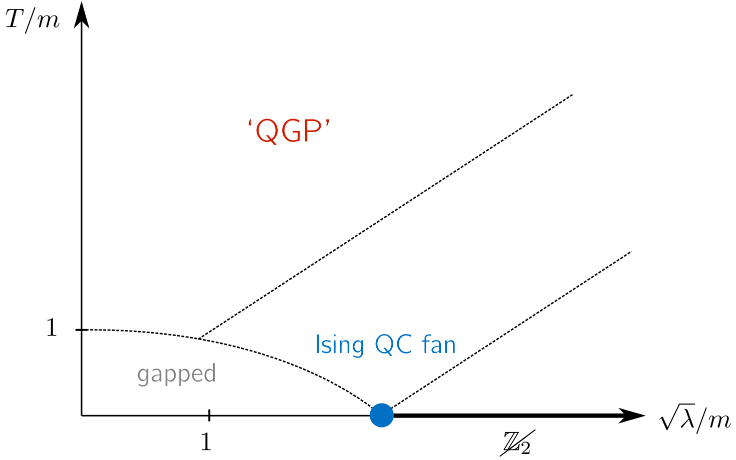
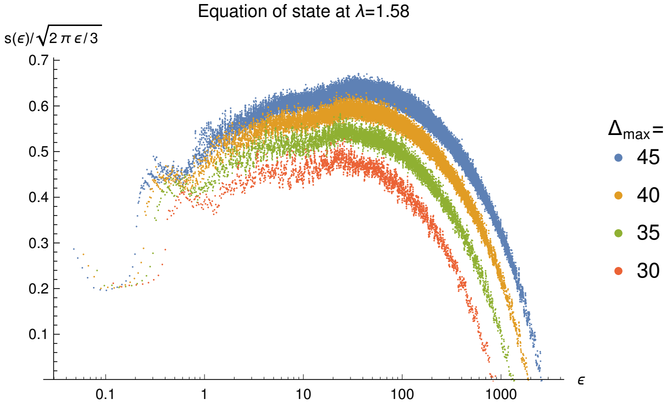


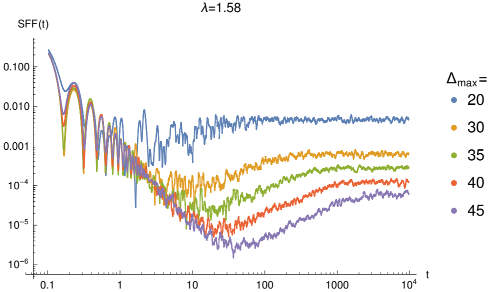
Equation of state

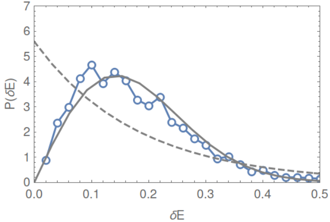
The entropy density is that of a free scalar $s = \sqrt{\frac{2}{3}\pi\epsilon}$

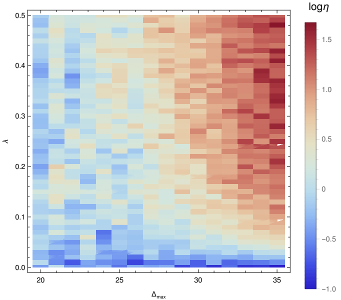


Access thermal physics microcanonically by using highly excited states

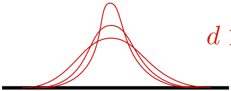
Some preliminary results:





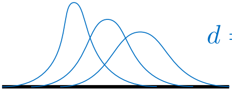


1. Brief Intro to Hydro
2. Weak fluctuations



$d > 0$

3. Strong fluctuations



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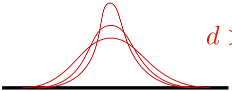

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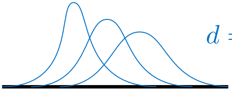

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Thanks!

LVD@uchicago.edu