Stress tensor sector of Holographic CFTs

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Motivation

$\text{AdS}_{d+1}/\text{CFT}_d$ **correspondence:**

- Gravity in $(d+1)$-dimensional asymptotically Anti de-Sitter spacetime.
- Conformal Field Theory in d-dimensional spacetime.

- Weakly coupled gravity is dual to strongly coupled CFT.
- Can we use CFTs to **define** and **describe** quantum gravity?
- First, we need to understand the duality at classical level in gravity.
- Second, learn more about strongly coupled CFTs.

(Very brief) Introduction to $\mathrm{CFTs}_{d>2}$

- CFTs are quantum field theories with conformal symmetry.
- Conformal symmetry $SO(d,2)$: Poincaré symmetry (P_μ, $M_{\mu\nu}$) + dilatation D + special conformal transformations K_μ:

$$D: x^\mu \to \lambda x^\mu, \quad \lambda > 0,$$

$$[D, K_\mu] = -iK_\mu, \qquad [D, P_\mu] = iP_\mu, \qquad [K_\mu, P_\nu] = i(\eta_{\mu\nu} D - M_{\mu\nu}).$$

- Local operators in CFT make representations of conformal group.
- **Highest weight representation:** primary operator $\mathcal{O}_s(x)$ + descendants $P_{\mu_1} \dots P_{\mu_k} \mathcal{O}_s(x)$.

$$[D, \mathcal{O}_s(0)] = i\Delta_{\mathcal{O}} \mathcal{O}_s(0), \qquad [K_\mu, \mathcal{O}_s(0)] = 0.$$

(Very brief) Introduction to $\mathrm{CFTs}_{d>2}$

- Conformal symmetry fixes one- and two-point corr. functions

$$\langle \mathcal{O}(x) \rangle = 0, \qquad \langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}},$$

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(y) \rangle = \frac{C_T}{|x-y|^{2d}} \mathcal{I}_{\mu\nu,\rho\sigma}(x-y).$$

- Convergent operator product expansions (OPE) in CFT:

$$\langle \mathcal{O}_1(x) \mathcal{O}_2(y) \ldots \rangle = \sum_{\Delta,J} \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_{\Delta,J}} \Pi(x-y, \partial_y) \langle \mathcal{O}_{\Delta,J}(y) \ldots \rangle.$$

- Dynamical data of conformal field theory: **spectrum** and **OPE coefficients**: $\{(\Delta, J), \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3}\}$.

Holographic CFTs

- Large central charge: $C_T \propto N^2 \to \infty$

  Example: If theory has $SU(N)$ gauge symmetry, the rank of gauge group N should be large.

- Conformal dimensions of "single-trace", primary operators with spin $s > 2$ should be large, $\Delta_{\text{gap}} \to \infty$.

  "single-traces": $\mathcal{O}_1, \mathcal{O}_2, \ldots, J_\mu, \ldots T_{\mu\nu}$,
  "double-traces": $[\mathcal{O}_{1,s_1}, \mathcal{O}_{2,s_2}]_{n,l} =: \mathcal{O}_{1,s_1} \partial^{2n} \partial_1 \ldots \partial_l \mathcal{O}_{2,s_2} :$.
  "multi-traces": $[\mathcal{O}_{1,s_1}, \ldots, \mathcal{O}_{k,s_k}]_{n,l} =: \mathcal{O}_{1,s_1} \ldots \mathcal{O}_{k-1,s_{k-1}} \partial^{2n} \partial_1 \ldots \partial_l \mathcal{O}_{k,s_k} :$.

- Holographic CFTs ($C_T \to \infty$ and $\Delta_{\text{gap}} \to \infty$) have a weakly coupled gravity dual with local physics below the AdS scale. [Heemskerk, Penedones, Polchinski, Sully, '09.]

Holographic CFTs

- We are interested in CFTs with large central charge $C_T \to \infty$ and large gap $\Delta_{\rm gap} \to \infty$. We study

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z,\bar{z})\mathcal{O}_H(0)\rangle,$$

where $\Delta_L \sim \mathcal{O}(1)$ and $\Delta_H \sim \mathcal{O}(C_T)$ while $\mu \sim \frac{\Delta_H}{C_T}$ is fixed and used as an expansion parameter.

- In CFT_2 there is an infinite-dimensional Virasoro algebra that strongly constraints correlators in $C_T \propto c \to \infty$ limit.

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^2-1)n\delta_{m+n,0}, \qquad n,m \in \mathbb{Z}$$

- $\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle\Big|_{V.v.b.} \sim \mathcal{G}_{d=2}(z,\mu,\Delta_L) = e^{\Delta_L \mathcal{F}(\mu,z)}$,
where $\mathcal{G}$ and $\mathcal{F}$ are known in closed form, that can be expanded as $\mathcal{G}_{d=2}(\mu,\Delta_L,z) = \sum_{k=0}^{\infty} \mu^k \mathcal{G}^{(k)}_{d=2}(\Delta_L,z)$. [Fitzpatrick, Kaplan, Walters, , '14.]

$\text{CFT}_{d=2}$

- At $\mathcal{O}\left(\mu^k\right)$, $\mathcal{G}^{(k)}_{d=2}(\Delta_L, z)$ can be written as

$$\mathcal{G}^{(k)}_{d=2}(\Delta_L, z) = \sum_{\{i_p\}} a_{i_1 \dots i_k} f_{i_1}(z) \dots f_{i_k}(z), \qquad \sum_{p=1}^{k} i_p = 2k,$$

where i_p are integers and

$$f_a(z) = (1-z)^a {}_2F_1(a, a, 2a, 1-z).$$

- $\mathcal{G}^{(k)}$ contains contributions from all operators made of k stress-tensors:

$$\begin{aligned} \mathcal{G}^{(0)}(z) &\sim 1 && - && \hat{1}, \\ \mathcal{G}^{(1)}(z) &\sim a_2 f_2 && - && T(z), \\ \mathcal{G}^{(2)}(z) &\sim a_{22} f_2^2 + a_{13} f_1 f_3 && - && :T(z)\partial \dots \partial T(z):. \end{aligned}$$

$\mathrm{CFT}_{d>2}$

- **No** Virasoro symmetry anymore.
- We consider the exchange of multi-stress tensors.
- Their contribution to the correlator we denote as stress-tensor sector:

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z,\bar{z})\mathcal{O}_H(0)\rangle\big|_{\mathrm{multi-stress\ tensors}} = \sum_k \mu^k \mathcal{G}^{(k)}(z,\bar{z}).$$

- Now, multi-stress tensors are labeled by their spin s and twist τ,

$$\tau = \Delta - s.$$

- Multi-stress tensors made with k stress-tensors contribute to $\mathcal{G}^{(k)}$.

Minimal-twist contributions in $\mathrm{CFT}_{d>2}$

- First, we focus on minimal-twist multi-stress tensor operators.
- Operators with minimal twist, that contribute at $\mathcal{O}\left(\mu^k\right)$, can be schematically represented as:

$$[T^{(k)}]_{\tau,s} =: T_{\mu_1\nu_1} \dots T_{\mu_{k-1}\nu_{k-1}} \partial_{\alpha_1} \dots \partial_{\alpha_l} T_{\mu_k\nu_k} :,$$

$$\tau = k(d-2),$$
$$s = 2k + l.$$

- The contribution of operators with the minimal twist is dominant over those from higher-twist operators in the lightcone limit $1 - \bar{z} \ll 1$.
- OPE coefficients of these operators are the same in all holographic CFTs. [Fitzpatrick, Huang, '19.][Fitzpatrick, Huang, Meltzer, Perlmutter, Simmons-Duffin, '20.]

Minimal-twist contributions in $\mathrm{CFT}_{d>2}$

- In even-dimensional spacetime, we propose

$$\mathcal{G}^{(k)}(z,\bar{z}) \underset{\bar{z}\to 1}{\approx} \frac{(1-\bar{z})^{k(\frac{d-2}{2})}}{[(1-z)(1-\bar{z})]^{\Delta_L}} \sum_{\{i_p\}} a_{i_1\ldots i_k} f_{i_1}(z)\ldots f_{i_k}(z),$$

$$i_p \in \mathbb{N}, \quad \sum_{p=1}^{k} i_p = k\left(\frac{d+2}{2}\right) = \frac{\tau}{2} + s_{min},$$

where s_{min} is the minimal spin of operators that contribute at $\mathcal{O}(\mu^k)$ (For minimal-twist operators: $\tau = k(d-2)$, $s_{min} = 2k$).

- We now have to fix the unknown coefficients $a_{i_1\ldots i_k}$.

  These can be fixed via the lightcone bootstrap.

Lightcone bootstrap in CFT_4

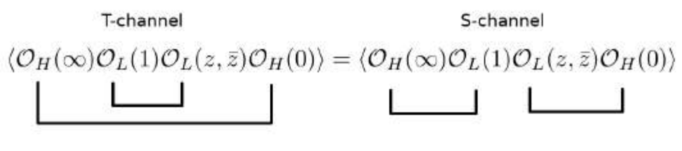

Figure: Lightcone bootstrap - schematically

$$\mathcal{G}(z,\bar{z}) = \frac{1}{[(1-z)(1-\bar{z})]^{\Delta_L}} \sum_{\mathcal{O}_{\tau,s}} P^{(HH,LL)}_{\mathcal{O}_{\tau,s}} g^{(0,0)}_{\tau,s}(1-z,1-\bar{z}),$$

where $P^{(HH,LL)}_{\mathcal{O}_{\tau,s}} \propto \lambda_{HH\mathcal{O}_{\tau,s}}\lambda_{LL\mathcal{O}_{\tau,s}}$ and $\mathcal{O}_{\tau,s} \in \{1, T, :T\Box^n\partial^l T:, \dots\}$.

$$\mathcal{G}(z,\bar{z}) = \frac{1}{(z\bar{z})^{\frac{1}{2}(\Delta_H+\Delta_L)}} \sum_{\mathcal{O}_{\tau',s'}} P^{(HL,HL)}_{\mathcal{O}_{\tau',s'}} g^{(\Delta_{HL},-\Delta_{HL})}_{\tau',s'}(z,\bar{z}),$$

where $P^{(HL,HL)}_{\mathcal{O}_{\tau,s}} \propto \lambda_{HL\mathcal{O}_{\tau',s'}}\lambda_{HL\mathcal{O}_{\tau',s'}}$ and $\mathcal{O}_{\tau',s'} \in \{: \mathcal{O}_H\Box^n\partial^l\mathcal{O}_L :\}$.

Lightcone bootstrap in CFT_4

T-channel

- Conformal blocks of operators in the T-channel:

$$g^{(0,0)}_{\tau,s}(1-z,1-\bar{z}) = (1-\bar{z})^{\frac{\tau}{2}}\left(f_{\frac{\tau}{2}+s}(z) + \mathcal{O}(1-\bar{z})\right).$$

- At $\mathcal{O}(\mu^0)$, there is only the identity contribution (disconnected correlator).
- At $\mathcal{O}(\mu)$, there is only the stress-tensor contribution that is fixed by the Ward identity:

$$\mathcal{G}^{(1)}(z,\bar{z}) = \frac{1}{((1-z)(1-\bar{z}))^{\Delta_L-1}}\frac{\Delta_L}{120(z-\bar{z})}\left(f_3(z)+f_3(\bar{z})\right)$$
$$\underset{\bar{z}\to 1}{\approx} \frac{(1-\bar{z})}{((1-z)(1-\bar{z}))^{\Delta_L}}\frac{\Delta_L}{120}f_3(z)$$

Lightcone bootstrap in CFT_4

S-channel: $\mathcal{O}_H \partial^{2n} \partial^l \mathcal{O}_L$

$$g^{(\Delta_{HL},-\Delta_{HL})}_{\Delta_H+\Delta_L+2n+\gamma,l}(z,\bar{z}) \underset{\Delta_H\to\infty}{\approx} \frac{(z\bar{z})^{\frac{1}{2}(\Delta_H+\Delta_L+2n+\gamma)}}{\bar{z}-z}\left(\bar{z}^{l+1}-z^{l+1}\right)$$

- OPE coefficients at $\mathcal{O}(\mu^0)$ fixed by the mean field theory $P^{(HL,HL);\mathrm{MFT}}_{n,l}$.
- At higher orders in μ, OPE coefficients and conformal dimensions get corrected.

$$\gamma_{n,l} = \sum_{k=1}^{\infty} \mu^k \gamma^{(k)}_{n,l} = \sum_{k=1}^{\infty} \mu^k \sum_{p=0}^{\infty} \frac{\gamma^{(k,p)}_n}{l^{k+p}},$$

$$P^{(HL,HL)}_{n,l} = P^{(HL,HL);\mathrm{MFT}}_{n,l} \sum_{k=0}^{\infty} \mu^k P^{(HL,HL);(k)}_{n,l}, \quad P^{(HL,HL);(k)}_{n,l} = \sum_{p=0}^{\infty} \frac{P^{(k,p)}_n}{l^{k+p}}.$$

Lightcone bootstrap in CFT_4

- $f_i(z) = q_{i,1}(z) + \log(z) q_{i,2}(z)$, where $q_{i,1/2}(z)$ are rational functions.
- By matching $\mathcal{O}(\mu)$ from S-channel with T-channel (stress-tensor contribution), we fix $\gamma_n^{(1,p)}$ and $P_n^{(1,p)}$.
- At $\mathcal{O}(\mu^2)$, terms that contain $\log^2(z)$ in the S-channel are fixed by $\gamma_n^{(1,p)}$ and $P_n^{(1,p)}$.
- Generally, at $\mathcal{O}(\mu^k)$, terms that contain $\log^i(z)$, $2 \leq i \leq k$, in the S-channel are fixed by OPE data up to $\mathcal{O}(\mu^{k-1})$.

Lightcone bootstrap in CFT_4

- On the other hand, ansatzes in the T-channel are of the following form:

$$\mathcal{G}^{(k)}(z,\bar{z}) \underset{\bar{z}\to 1}{\approx} \frac{(1-\bar{z})^k}{((1-z)(1-\bar{z}))^{\Delta_L}} \Big(\tilde{q}_k(z, a_{i_1 \dots i_k}) \log^k(z) + \dots \\ + \tilde{q}_1(z, a_{i_1 \dots i_k}) \log(z) + \tilde{q}_0(z, a_{i_1 \dots i_k})) \Big),$$

where $\tilde{q}_i(z, a_{i_1 \dots i_k})$ are rational functions of their arguments.

- Therefore, we can match terms that behave as $\log^i(z)$ for $2 \leq i \leq k$, with those from the S-channel calculation, that are fixed in terms of OPE data at subleading order in μ.

Lightcone bootstrap in CFT_4

- At $\mathcal{O}(\mu^2)$, we match terms that contain $\log^2(z)$ from T- and S-channel.
- Matching these terms fixes coefficients the unknown coefficients a_{33}, a_{24} and a_{15} in the ansatz:

$$\mathcal{G}^{(2)}(z,\bar{z}) \underset{\bar{z}\to 1}{\approx} \frac{(1-\bar{z})^2}{[(1-z)(1-\bar{z})]^{\Delta_L}} \left(\frac{\Delta_L}{28800(\Delta_L-2)}\right) \times \left\{(\Delta_L-4)(\Delta_L-3)f_3^2(z) + \frac{15}{7}(\Delta_L-8)f_2(z)f_4(z) + \frac{40}{7}(\Delta_L+1)f_1(z)f_5(z)\right\}$$

- By this means, we reproduce the correlator calculated in [Kulaxizi, Ng, Parnachev, '19.].
- Same method has been used calculate $\mathcal{O}(\mu^4)$ contributions in $d=4$ and $\mathcal{O}(\mu^2)$ in $d=6$. [Karlsson, Kulaxizi, Parnachev, PT, '19.]

Non-minimal twist - CFT_4

[Karlsson, Kulaxizi, Parnachev, PT, '20.]

- In $d = 4$, at $\mathcal{O}(\mu^2)$, minimal twist is $\tau = 4$, while first non-minimal twist double stress tensors have twist $\tau = 6$.

- These are two families of such operators:

$$: T_{\mu\alpha}\partial_{\lambda_1}\dots\partial_{\lambda_s}T^{\alpha}{}_{\nu} :, \qquad \frac{\tau}{2} + s_{min} = 5,$$

$$: T_{\mu\nu}\partial_{\lambda_1}\dots\partial_{\lambda_s}\partial^2 T_{\rho\sigma} :, \qquad \frac{\tau}{2} + s_{min} = 7.$$

- Now, we propose:

$$\mathcal{G}^{(2,1)}(z,\bar{z}) \underset{\bar{z}\to 1}{\propto} \frac{(1-\bar{z})^3}{((1-z)(1-\bar{z}))^{\Delta_L}}\Big(b_{14}f_1f_4 + b_{23}f_2f_3 + c_{16}f_1f_6 + c_{25}f_2f_5 + c_{34}f_3f_4\Big).$$

Non-minimal twist - CFT_4

[Karlsson, Kulaxizi, Parnachev, PT, '20.]

- Again, we use the lightcone bootstrap to fix the unknown coefficients.
- We look for terms proportional to the $\log^2(z)$ in the S-channel calculation.
- We have to keep subleading corrections to the S-channel OPE data in large-spin limit.
- We get $b_{23}, c_{16}, c_{25}, c_{34}$ in terms of Δ_L and b_{14}.
- b_{14} is the OPE coefficient of $: T_{\mu\alpha} T^{\alpha}{}_{\nu} :$.
- **Generally:** The lightcone bootstrap does not fix the OPE coefficients of operators with spin $s = 0, 2$.

Exponentiation and OPE coefficients

- It is shown that one can write the minimal-twist stress-tensor sector up to the $\mathcal{O}(\mu^4)$ in $d = 4$ as

$$\mathcal{G}(z, \bar{z}) \underset{\bar{z}\to 1}{\approx} \frac{1}{[(1-z)(1-\bar{z})]^{\Delta_L}} e^{\Delta_L \mathcal{F}(\mu; z, \bar{z})},$$

  for some function $\mathcal{F}$ which is a rational function of Δ_L and remains $\mathcal{O}(1)$ as $\Delta_L \to \infty$.

- One can use the following relation

$$f_a(z) f_b(z) = \sum_{m=0}^{\infty} p[a, b, m] f_{a+b+2m}(z)$$

  to read off the OPE coefficients of minimal-twist multi-stress tensors from ansatzes with fixed coefficients.

Universality beyond the large gap

[Karlsson, Parnachev, PT, 20xx.xx]

- We study free theory with large central charge, $C_T \to \infty$, $\Delta_{\rm gap} \to 0$.
- Holographic OPE coef.: $P^{(HH,LL)}_{[T^2]_{4,4}} = \mu^2 \left(\frac{7\Delta_L^3 + 6\Delta_L^2 + 4\Delta_L}{201600(\Delta_L - 2)} \right)$.
- Free theory OPE coef.: $P^{(HH,LL)}_{[T^2]_{4,4}} = \mu^2 \left(\frac{7{\Delta_L}^2 - 7\Delta_L}{201600} \right)$.
- In the limit $1 \ll \Delta_L \ll \Delta_H \lesssim C_T$, these two are the same!

- Stress-tensor sector of the theory thermalizes

$$\langle T_{\mu\nu} \rangle_{S^1 \times S^{d-1}} = \langle \mathcal{O}_H T_{\mu\nu} \mathcal{O}_H \rangle_{\mathbb{R}^1 \times S^{d-1}} \implies \mu = \frac{8}{3} \frac{\pi^4 R^4}{\beta^4},$$

$$\langle [T^{(k)}]_{\tau,s} \rangle_{S^1 \times S^{d-1}} = \langle \mathcal{O}_H [T^{(k)}]_{\tau,s} \mathcal{O}_H \rangle_{\mathbb{R}^1 \times S^{d-1}}.$$

Conclusions and future developments

- We found a method for the efficient calculation of the multi-stress tensor contributions.
- We confirm the universality of OPE coefficients of minimal-twist multi-stress tensors.
- We find that the minimal-twist contributions exponentiate in analogy with the Virasoro vacuum block.

In future:

- What happens with minimal-twist OPE coefficients in theory with finite Δ_{gap}?[Fitzpatrick, Huang, Meltzer, Perlmutter, Simmons-Duffin, ′20.], [20xx.xx - Karlsson, Parnachev, PT]
- Exploring the possibility of summing all minimal-twist contributions in a closed analytic form.

THANK YOU.