$T\bar{T}$-deformed CFTs out of equilibrium

M. Medenjak, G. Policastro, T. Yoshimura, arXiv:2011.05827, arXiv:2010.15813

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Motivation

- Understanding the corrections to the Stephan-Boltzmann law, if T is not traceless
- The effects of CFT breaking: leading order correction if Lorenz invariance is unbroken (no umklapp processes)
- Connecting holography and GHD results

Overview

- Bipartition protocol and transport
- $T\bar{T}$ deformed CFTs
  - Integrability
  - holography
- Integrability/GHD approach
  - GHD overview
  - NESS
  - Energy/momentum Drude weights
  - Diffusion constants
- Holographic approach
  - NESS
  - Drude weights

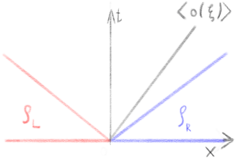
Bipartition protocol

- Bipartition protocol: two sides prepared in different stationary states

$$\rho(0) = \rho_L \otimes \rho_R$$

- Nonequilibrium steady state (NESS) forms along the light rays $\xi = \frac{x}{t}$ for large times $t \to \infty$:

$$\lim_{t\to\infty} \langle o(\xi t, t)\rangle = \langle o(\xi)\rangle_{NESS}$$



- What is NESS for $T\bar{T}$ deformed CFTs if two boosted thermal states $\rho_{L/R} \propto \exp(-\beta_{L/R} H + \nu_{L/R} P)$ are joined?

Transport coefficients

- Drude weight D_{ij}, the Onsager coefficient $\mathfrak{L}_{ij}^{(\text{reg})}$, and diffusion constant $\mathcal{D}_{ij}$ (response of the current j_i to the charge q_j)

$$\sigma_{ij}(\omega) = D_{ij}\delta(\omega) + \sigma_{ij}^{(\text{reg})}(\omega), \quad \mathfrak{L}_{ij} = \sigma_{ij}^{(\text{reg})}(0), \quad \mathfrak{L}_{ij} = \mathfrak{D}_{ik} C_j^k$$

- Linear response expressions

$$D_{ij} = \lim_{t\to\infty} \int_{-t}^{t} \frac{\mathrm{d}s}{2t} \int_{\mathbb{R}} \mathrm{d}x \langle j_i(x,s) j_j(0,0)\rangle^c$$

$$\mathfrak{L}_{ij} = \int_{\mathbb{R}} \mathrm{d}t \left(\int_{\mathbb{R}} \mathrm{d}x \langle j_i(x,t) j_j(0,0)\rangle^c - D_{ij} \right)$$

- From the bipartition protocol we can extract transport coefficients by considering infinitesimal difference of chemical potentials

$$D_{ij} = \lim_{t\to\infty} \frac{1}{2t} \frac{\int_{\mathbb{R}} \mathrm{d}x \, \langle j_i(x,t)\rangle_{\delta\mu_j}}{\delta\mu_j}$$

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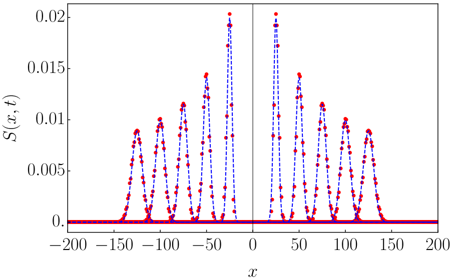
Transport coefficients



$T\bar{T}$ deformation

- $T\bar{T}$ deformation of the theory is obtained via the set of infinitesimal changes

$$\mathcal{L}^{(\sigma+\delta\sigma)} = \mathcal{L}^{(\sigma)} + \frac{\delta\sigma}{2}\det T_{\mu\nu}$$

- The theory preserves Lorenz invariance. If we know the spectrum of the undeformed theory, we can solve the deformed theory

F.A. Smirnov and A.B. Zamolodchikov, Nucl. Phys. B 915 (2017) 363

$$\partial_\sigma E_n(R,\sigma) = E_n(R,\sigma)\partial_R E_n(R,\sigma) + \frac{1}{R}P_n^2(R)$$

- On the level of the S matrix the deformation is induced by the multiplication by a phase factor

$$S^{(\sigma)}(\theta) = e^{\mathrm{i}\Sigma(\theta)}S^{(0)}(\theta), \quad \Sigma(\theta) = -\sigma p_+(\theta_1)p_-(\theta_2)$$

$p_\pm(\theta) = \pm\frac{M}{2}e^{\pm\theta}$

Deformed correspondence

- Gravity in 3d in radial coordinates (ρ-radius)

$$ds^2 = \ell^2 \frac{\mathrm{d}\rho}{4\rho} + \left(\frac{g^{(0)}_{\alpha\beta}(x^\alpha)}{\rho} + g^{(2)}_{\alpha\beta}(x^\alpha) + \rho g^{(4)}_{\alpha\beta}(x^\alpha) \right) dx^\alpha dx^\beta$$

  ($g^{(4)}$ is related to $g^{(2)}$)
- Undeformed case asympthotically ($\rho \to 0$): $g^{(2)} = 8\pi G \ell \hat{T}$ and $g^{(0)} = \gamma_0$
- There is a nonlinear connection between the deformed and undeformed stress energy tensor and metric which gives us

$$\gamma_\mu = g^{(0)} + \mu g^{(2)} + \mu^2 g^{(4)}$$
$$\hat{T}_\mu = \frac{1}{8\pi G \ell}(g^{(2)} + 2\mu g^{(4)})$$

  with $\mu = -\frac{c\sigma}{12\pi\ell^2}$ and $\hat{T}_{\alpha\beta} = T_{\alpha\beta} - \gamma_{\alpha\beta} T^\gamma_\gamma$. γ_μ is exactly the induced metric at $\rho = \mu$.

M Guica, R Monten - arXiv preprint arXiv:1906.11251

L. McGough, M. Mezei and H. Verlinde, JHEP 04 (2018) 010

Deformed correspondence

CFT in flat metric

$$ds^2 = \ell^2 \frac{d\rho^2}{4\rho^2} + \frac{dudv}{\rho} + \mathcal{L}(u)\, du^2 + \bar{\mathcal{L}}(v)\, dv^2 + \rho \mathcal{L}(u)\bar{\mathcal{L}}(v)\, dudv\,.$$

Deformed metric at the boundary $\gamma_\mu = dUdV$ is flat in new coordinates

$$U = u + \mu \int^v \bar{\mathcal{L}}(v')dv'\,, \quad V = v + \mu \int^u \mathcal{L}(u')du'$$

where $\mathcal{L}$ and $\bar{\mathcal{L}}$ are constant for a BTZ black hole and correspond to the finite temperature and momentum in the deformed CFT:

$$\beta = \frac{\pi\ell}{2}\left(\frac{1}{\sqrt{\bar{\mathcal{L}}}} + \frac{1}{\sqrt{\mathcal{L}}}\right)(1 - \mu\sqrt{\mathcal{L}\bar{\mathcal{L}}}), \quad \nu = \frac{\mathcal{L} - \bar{\mathcal{L}}}{\mathcal{L} + \bar{\mathcal{L}}}$$

Inverting the coordinate transformation and using the connection:

$$T_{tt} = \frac{1}{8\pi G\ell}\frac{\mathcal{L} + \bar{\mathcal{L}} - 2\mu\mathcal{L}\bar{\mathcal{L}}}{1 - \mu^2\mathcal{L}\bar{\mathcal{L}}}\; T_{xt} = \frac{1}{8\pi G\ell}\frac{\mathcal{L} - \bar{\mathcal{L}}}{1 - \mu^2\mathcal{L}\bar{\mathcal{L}}}$$

$$T_{xx} = \frac{1}{8\pi G\ell}\frac{\mathcal{L} + \bar{\mathcal{L}} + 2\mu\mathcal{L}\bar{\mathcal{L}}}{1 - \mu^2\mathcal{L}\bar{\mathcal{L}}}$$

Euler scale hydrodynamics

- We assume a separation of time scales (systems equilibrate locally). If we consider Euler (ballistic) scales $\xi = \frac{x}{t}$, $\lim_{t\to\infty}$, then the expectation values of local observables depend only on modulation of chemical potentials

$$\bar{o}_i(x,t) = \lim_{\ell\to\infty} \langle o(\ell x, \ell t)\rangle = \langle o \rangle_{\{\beta_i(x,t)\}}$$

- This implies that the dynamics of charges is governed purely by the set of continuity equation

$$\partial_t \bar{q}_i(x,t) + \partial_x \bar{j}_i(x,t) = 0$$

since $\bar{o}_i(x,t)$ depend only on the set of $\{\beta_i(x,t)\}$, or equivalently $\bar{q}_i(x,t)$. Generalized hydrodynamics $\to$ a prescription how to obtain averages of currents.

Thermodynamics of integrable systems

- In order to obtain $\bar{j}_i(x,t)$ we need to discern the expectation values of currents and charges in ρ.

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- Integrable systems have a stable quasi-particle structure. Upon scattering the set of momenta is preserved, and particles undergo phase shifts. Thermodynamics is governed by pseudoenergy $\varepsilon(\theta)$ (θ is a rapidity variable)

$$\varepsilon(\theta) = w(\theta) + \int \mathrm{d}\theta' \, T(\theta,\theta') L(\varepsilon(\theta'))$$

$w(\theta) = \sum \beta^i h_i(\theta)$–source term ($\rho \sim exp(\sum \beta^i Q_i)$), $T(\theta,\theta')$–two body phase shift, $h_i(\theta)$ the amount of charge q_i carried by the quasiparticle θ, $L(\varepsilon)$–free energy function (fermions: $L = -\log(1+\exp(-\varepsilon))$)

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- Free energy f and occupation function $n(p)$

$$f = -\int \frac{\mathrm{d}\theta}{2\pi} p'(\theta) L(\varepsilon) \qquad n(p) = \frac{\mathrm{d}L(\epsilon)}{\mathrm{d}\varepsilon}$$

GHD: Currents and charges

- Expectation value of conserved charge $\overline{q}_i$ can of-course be obtained by taking the derivative of f wrt β_i

$$\overline{q}_i = \int \frac{\mathrm{d}p}{2\pi} n(p) h_i^{dr}(p) = \int \frac{\mathrm{d}p}{2\pi} \rho_p(p) h_i(p); \qquad h_i^{dr}(p) = \frac{\partial \epsilon(p)}{\partial \beta_i}$$

O. A. Castro-Alvaredo, B. Doyon, and T. Yoshimura Phys. Rev. X 6, 041065 (2016).

B. Bertini, M. Collura, J. De Nardis, and M. Fagotti Phys. Rev. Lett. 117, 207201 (2016).

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$$\bar{j}_i = \int \frac{\mathrm{d}p}{2\pi} E'(p) n(p) h_i^{dr}(p) = \int \mathrm{d}p v^{eff}(p) \rho_p(p) h_i(p)$$

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- GHD equation

$$\partial_t \rho_p(p,x,t) + \partial_x (v^{eff}(p,x,t) \rho_p(p,x,t)) = 0$$

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NESS in $T\overline{T}$-deformed CFT

- We consider $\rho \sim \exp(-\beta H + \nu P)$. $T\bar{T}$ deformation induces the L-R scattering $\tilde{T}_{\pm\mp}(\theta, \theta') = -\sigma p_+(\theta) p_-(\theta')/(2\pi) =$ two body phase shift

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$$\varepsilon_\pm(\theta) = \beta_\pm E_\pm(\theta) - T \star L_\pm(\theta) - \tilde{T}_{\pm\mp} \star L_\mp(\theta),$$
- Surprisingly $v^{eff}_\pm$ does not depend on θ and simply reads
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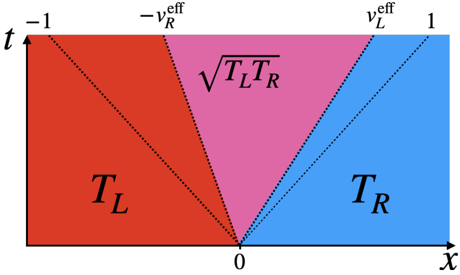
where $\rho_\pm$ energy densities associated with the L/R movers.
- Hydrodynamics is governed simply by two coupled equations
$$\partial_t \rho_\pm + \partial_x (v^{eff}_\pm \rho_\pm) = 0.$$
- Solution ($e_{RL} = 1/(1 - \left(\frac{\pi\sigma c}{12}\right)^2 \tilde{T}_L^2 \tilde{T}_R^2)$)
$$\langle j_E \rangle_{\mathrm{NESS}} = \frac{\pi c}{12} e_{RL} \left(\tilde{T}_L^2 - \tilde{T}_R^2 \right),$$
$$\langle j_P \rangle_{\mathrm{NESS}} = \frac{\pi c}{12} e_{RL} \left(\tilde{T}_L^2 + \tilde{T}_R^2 - \frac{\pi c \sigma}{6} \tilde{T}_L^2 \tilde{T}_R^2 \right)$$
Similar to CFTs, but no chiral separation $\langle j \rangle_{\mathrm{NESS}} \neq f(\beta_L) - f(\beta_R)$

NESS corresponds to the thermal state with effective temperature $T = \sqrt{T_L T_R}$ with the boost $\tanh \nu = \frac{(\beta_L - \beta_R)}{(\beta_L + \beta_R)}$



NESS has the same form as in the CFT case (velocity in CFT is ± 1)

Transport coefficients

- Momentum and energy Drude weights are simple functions of energy density and pressure

(general expression: E. Ilievski and J. De Nardis Phys. Rev. Lett. 119, 020602)

$$D_{EE} = \frac{\mathtt{e}+\mathtt{p}}{\beta} = \frac{\pi c}{3 v_c} T^3, \; D_{PP} = \left(\frac{\mathtt{p}}{\mathtt{e}}\right)^2 D_{EE} = \frac{\pi c v_c}{3} T^3$$

and can be expressed in terms of the effective velocity

$$v^{\text{eff}} = \sqrt{1 - \frac{\pi \sigma c T^2}{3}}.$$

- These results are generalizations of CFT results which can be obtained by $v^{\text{eff}} \to 1$
- $T\bar{T}$ deformation **induces diffusion** (energy diffusion is absent because of Lorenz invariance)

(general expression: J. De Nardis, D. Bernard, and B. Doyon Phys. Rev. Lett. 121, 160603)

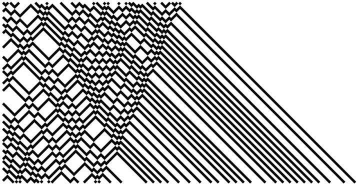
$$\mathfrak{L}_{PP} = \frac{\sigma^2}{2} v_c D_{EE}^2 = \frac{\sigma^2}{2} \frac{D_{PP}^2}{v_c^3}$$

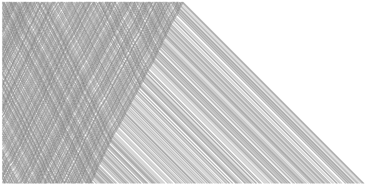
$T\overline{T}$-deformed CFT and RCA 54

- Euler scale hydrodynamics matches exactly the hydrodynamics of RCA 54 A. Bobenko, M. Bordemann, C. Gunn, and U. Pinkall, CMP 158, 127 (1993).

$$\partial_t \rho_\pm + \partial_x (v_\pm^{eff} \rho_\pm) = 0.$$

A. J. Friedman, S. Gopalakrishnan, and R. Vasseur, Phys. Rev. Lett. 123, 170603 (2019)





K. Klobas, M. Medenjak, T. Prosen, M. Vanicat CMP 371, 651–688 (2019)

Holographic solution

- Bipartition protocol is solved by joining the two solutions at time $t = 0$ (done in CFT by M. J. Bhaseen, B. Doyon, A. Lucas, K. Schalm Nature Physics 11, 509–514(2015))

$$\mathcal{L}(u) = \mathcal{L}_L \theta(-u) + \mathcal{L}_R \theta(u) \quad \bar{\mathcal{L}}(v) = \bar{\mathcal{L}}_L \theta(-v) + \bar{\mathcal{L}}_R \theta(v)$$

This means that NESS comprises two shock waves at $u = 0$ and $v = 0$. The coordinate transformation can be solved exactly

$$x = -\frac{1 + \mu \bar{\mathcal{L}}_L}{1 - \mu \bar{\mathcal{L}}_L} t \quad x = \frac{1 + \mu \mathcal{L}_R}{1 - \mu \mathcal{L}_R} t$$

- NESS current is

$$\langle j_E \rangle_{\mathrm{NESS}} = \frac{1}{8\pi G \ell} \frac{\mathcal{L}_L - \mathcal{L}_R}{1 - \mu^2 \mathcal{L}_L \mathcal{L}_R}$$

$$\langle j_P \rangle_{\mathrm{NESS}} = \frac{1}{8\pi G \ell} \frac{\mathcal{L}_L + \mathcal{L}_R + 2\mu \mathcal{L}_L \mathcal{L}_R}{1 - \mu^2 \mathcal{L}_L \mathcal{L}_R}$$

- We can reproduce the integrability result by identifying

$$\sqrt{\mathcal{L}} = \frac{\beta \left(\sqrt{\frac{4\pi^2 l^2 \mu}{\beta^2} + 1} - 1 \right)}{2\pi l \mu} = \frac{\pi \ell}{\tilde{\beta}_{\pm}(\beta, 0)}$$

Holographic approach

- The results hold at all orders in the deformation parameter
- No diffusion: while Onsager coefficient is finite, the broadening of the fronts is governed by diffusion. In holography diffusion is suppressed since $\mathfrak{D} = \frac{1}{C}\mathfrak{L}$ and $C \propto c$
- Momentum diffusion can be obtained in the leading order σ^2 also via the conformal perturbation theory, and agrees with the integrability calculation

Overview and Outlook

- Nonperturbative test of the deformed holographic correspondence
- Perfect agreement between holographic and integrable CFTs $\rightarrow$ Universal results
- Generalization to other deformations/GGEs/quantities (entanglement, operator spreading,...)
- Exact relation to cellular automata