Chiral hydrodynamics of plasma in strong magnetic fields

Seminar, University of Ljubljana

March 18th, 2021

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

[Ammon, Kaminski, Koirala, Leiber, Wu; JHEP (2017)]

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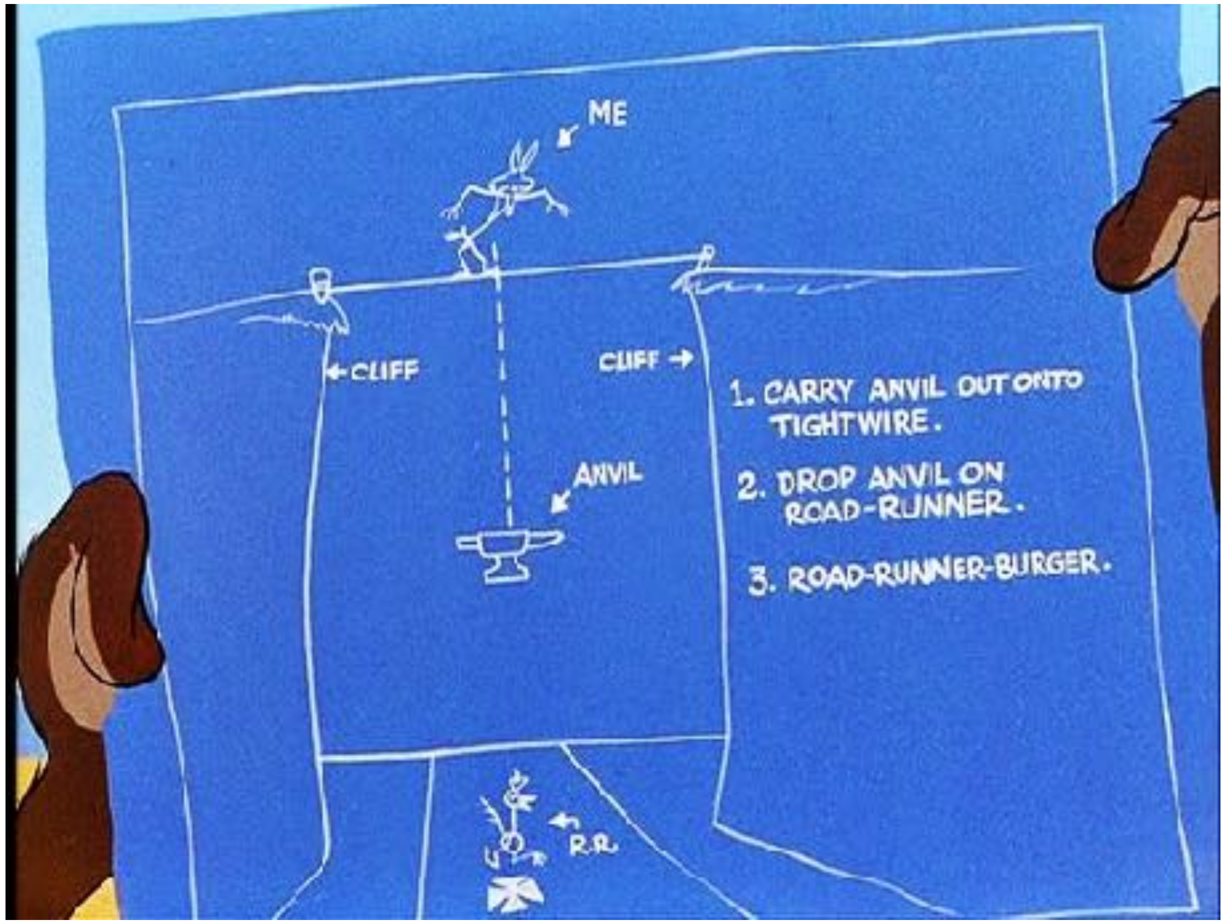
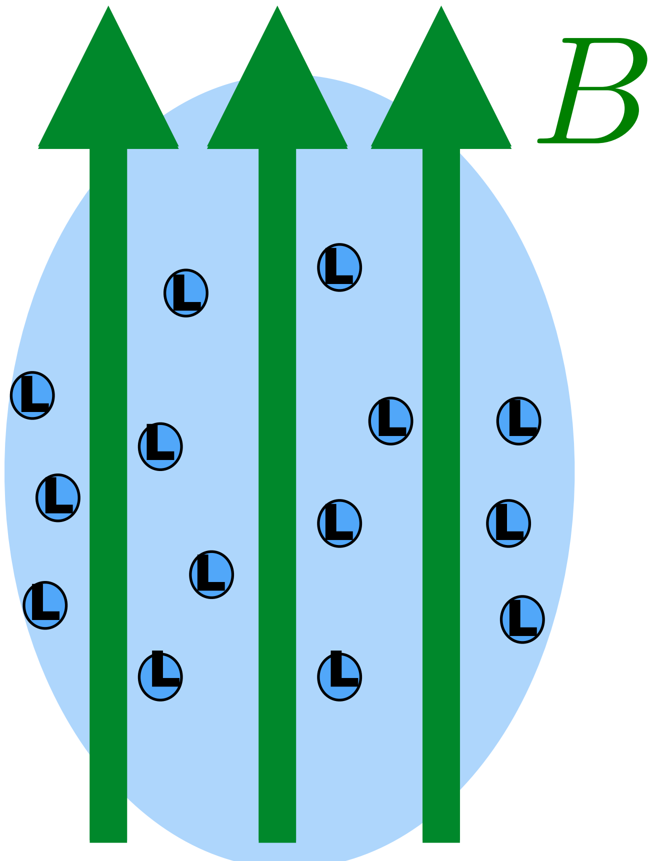
Outline



1. Hydrodynamics - Methods
2. Hydrodynamic results
   - Kubo formulae
   - novel transport effects
3. Holographic model & results
4. Discussion

Why should I not go and get a coffee?

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]



charged (3+1)-dimensional relativistic fluid of chiral fermions in *magnetic field*

Sneak preview of results

5 novel transport effects :

- ✦ 1 perpendicular magnetic vorticity susceptibility
- ✦ 1 shear-induced conductivity
- ✦ 2 expansion-induced conductivities
- ✦ **1 *non-dissipative: shear-induced Hall conductivity***

$$j_x \sim c_{10}(\partial_y v_z + \partial_z v_y)$$

2 Hall viscosities & modified Hall physics

Kubo formulae for >20 transport coefficients

Reminder: shear viscosity $\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$

Novel: expansion-induced conductivity $c_4 \sim \langle [j_z, T_{xx}] \rangle$

Why should I not go and get a coffee?

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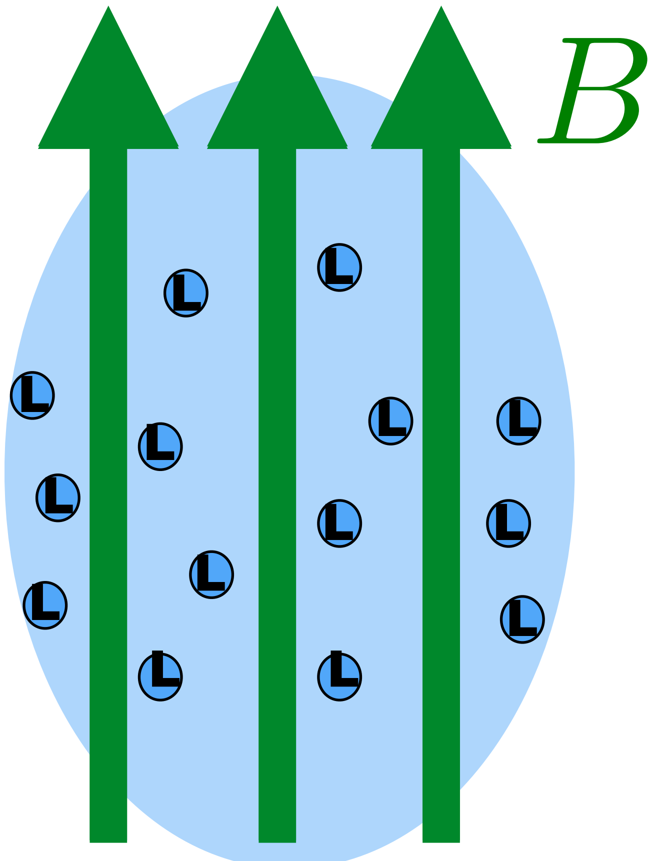

charged (3+1)-dimensional relativistic fluid of chiral fermions in magnetic field

➡**Also: I need your expertise.**

Sneak preview of results

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- ✦ **1 *non-dissipative:* *shear-induced Hall conductivity***

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Reminder:
shear viscosity

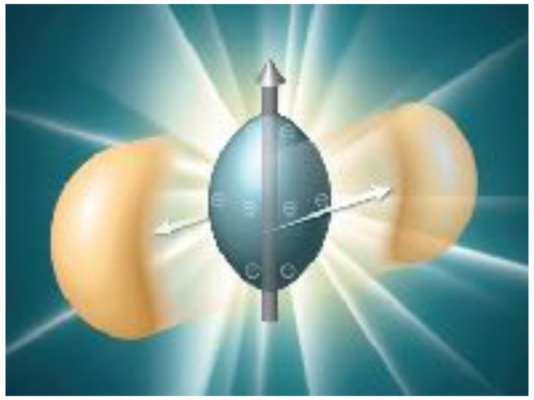
$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

Novel:
expansion-induced conductivity

$$c_4 \sim \langle [j_z, T_{xx}] \rangle$$

Quark Gluon Plasma

- strong magnetic field B
- chiral anomaly
- chiral transport effects, e.g. ***chiral magnetic effect / wave***

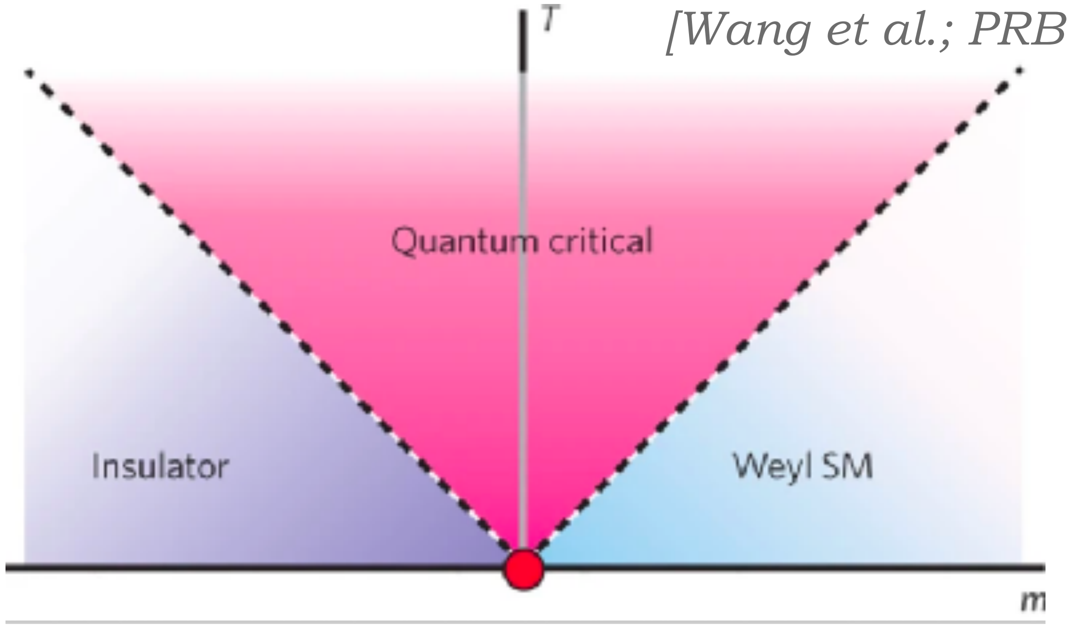


[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Kharzeev,McLerran,Warringa; Nucl.Phys.A (2008)]
[Kharzeev,Yee; PRD (2011)]

Weyl-semimetals

- relativistic Weyl fermions + B
- chiral anomaly, ***CME / wave***

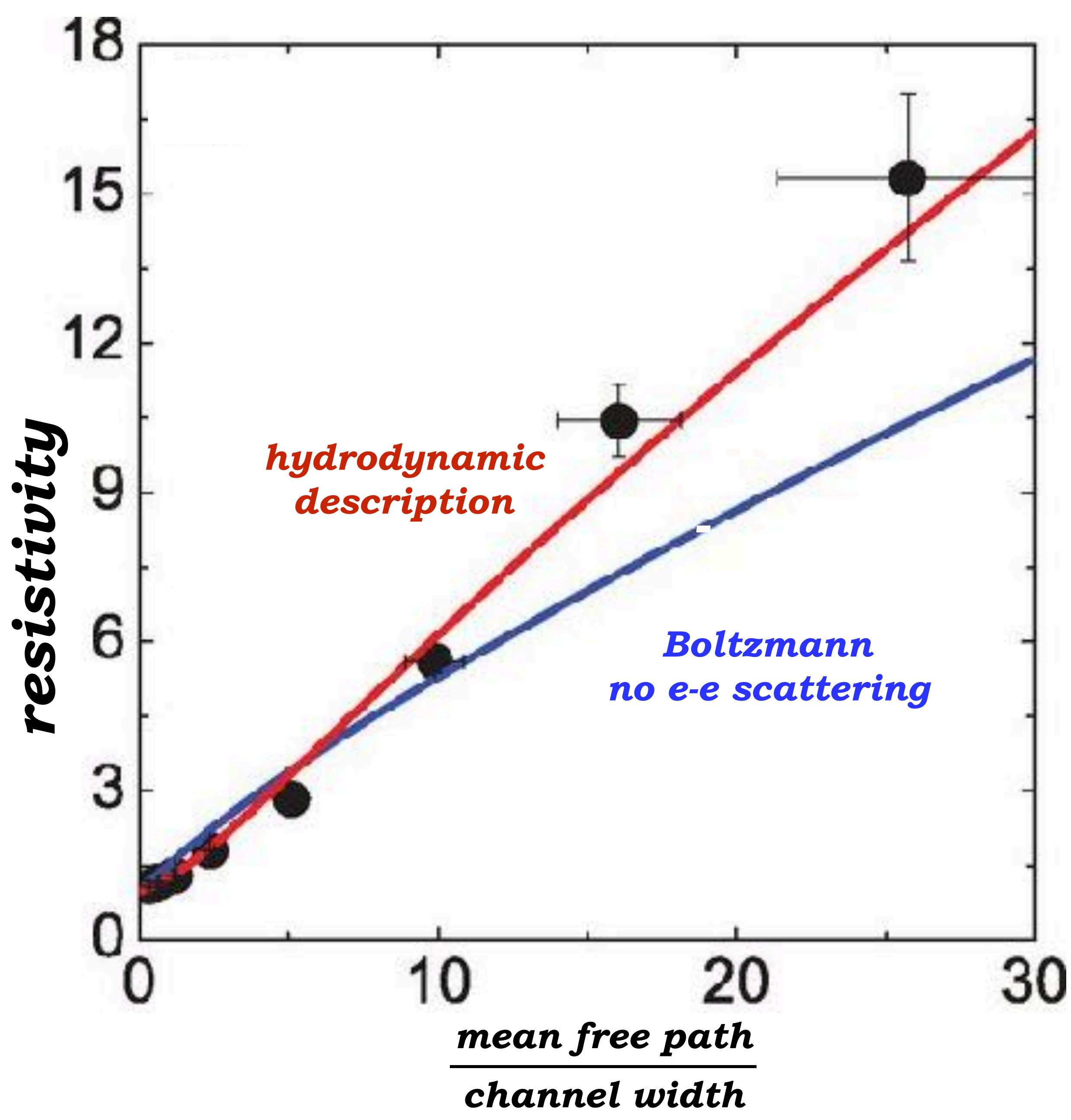
[Yang et al.; Nature (2014)]
[Wang et al.; PRB (2019)]



quantum critical point

magnetization

Hydrodynamics - Examples (2)



Electron fluids

[Molenkamp, de Jong; PRB (1994)]

- Mean free path for electron to scatter from another electron is small compared to other scatterings
- experimentally realized

Resistivity versus channel width in thin $PdCoO_2$ wires *[Moll et al.; Science (2016)]*

1. Hydrodynamics - Concepts

Which are the relevant quantities in systems near equilibrium, and how can we predict their behavior?

Hydrodynamics

- effective description of systems at late times and large distances
- conserved quantities survive
- small gradients
- large temperature

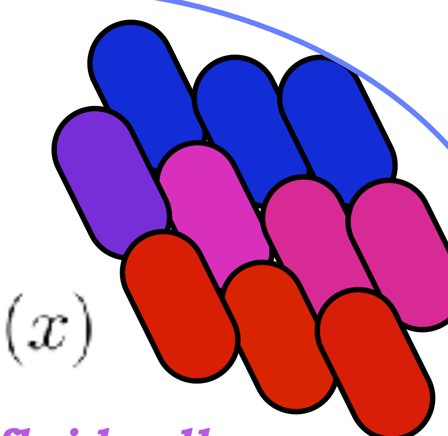
$$\partial_t\, e^{-i\omega t} = -i\omega\, e^{-i\omega t}$$

$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

In this presentation also :

$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$

$$T(t,\vec{x}) \equiv T(x)$$



fluid cells with distinct temperatures

1. Hydrodynamics - Formalism

Universal **effective field theory (EFT)**

[Baier, Romatschke, Romatschke, Son, Starinets, Stephan; JHEP (2008)]

- expansion in gradients of fields
- systematic construction
- generating functional
  [Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
  [JHEP (2011)]
  [Banerjee et al. JHEP (2012)]

[Previously: *[Landau, Lifshitz]* phenomenological]

Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields
- constitutive equations
- conservation equations
- sources
  [Luttinger]

1. Hydrodynamics - Formalism

Universal **effective field theory (EFT)**

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phenomenological]

Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields $T(x)$, $n(x)$, $u^\alpha(x)$

temperature   *charge density*   *fluid velocity*

- constitutive equations

$$\langle j^\alpha \rangle = n\, u^\alpha + \nu^\alpha$$

ideal hydro   *derivative corrections*

- conservation equations

$$\nabla_\alpha \langle j^\alpha \rangle = 0$$

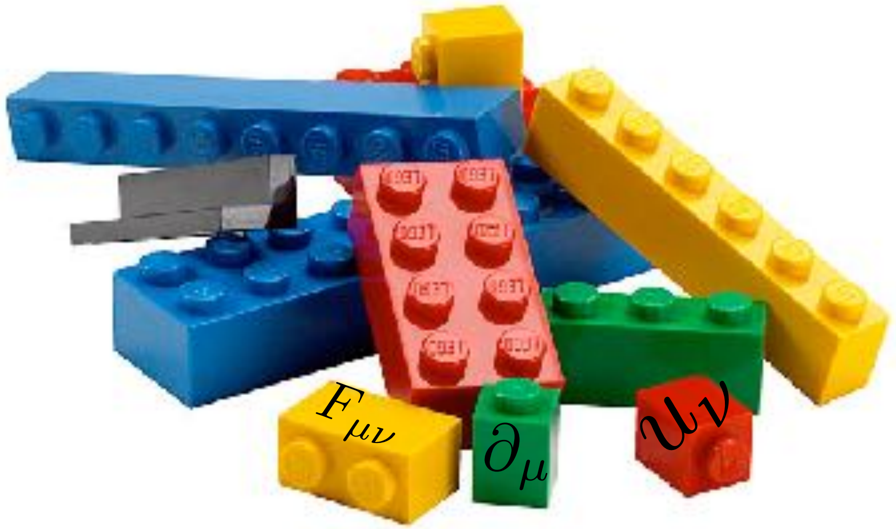
e.g. continuity: $\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$

- sources

[Luttinger] $g_{\alpha\beta}(x)$, $A_\alpha(x)$

metric   *gauge field*

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group



Examples: $\nabla_\nu u^\nu$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu \nabla_\lambda u_\rho$$

vorticity

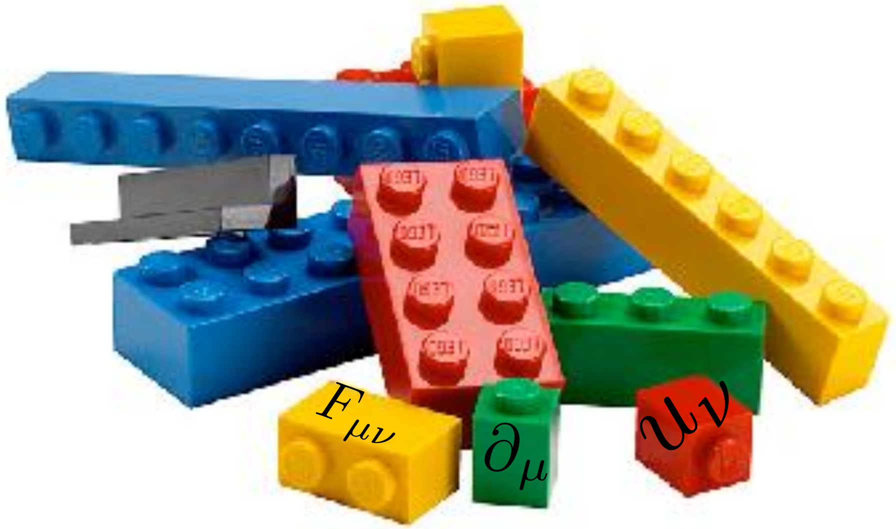
2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

3. **"Old school":** Further restricted by positivity of local entropy production: $\nabla_\mu J^\mu_s \geq 0$ *[Landau, Lifshitz]*

1. Hydrodynamics - Construction

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group



Examples: $\nabla_\nu u^\nu$

$$\underset{\text{vorticity}}{\omega^\mu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho$$

2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

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4. **Modern:** Construct generating functional, use field theory restrictions (Onsager relations, analyticity, Ward identities) *[Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)] [JHEP (2011)] [Banerjee et al. JHEP (2012)]*

➡ **General hydrodynamic 1-point functions**

[Ammon, Kaminski et al.; JHEP (2017); and arXiv (2020)]

Variation of the ...

... constitutive relations (1-point functions)

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

$$G^R_{T^{\mu\nu},J^\alpha} = \frac{\delta}{\delta A_\alpha} T^{\mu\nu}_{\text{on-shell}}[A,g]$$

$$G^R_{J^\mu T^{\alpha\beta}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\alpha\beta}} \left(\sqrt{-g}\, J^\mu_{\text{on-shell}}[A,g]\right)$$

Recall Onsager relations

$$G^R_{TJ} = \eta_T \eta_J G^R_{JT}$$

... equilibrium generating functional

$$\delta W_s[A,g] = \int d^4x \sqrt{-g} \left(\tfrac{1}{2} T^{\mu\nu}_{\text{eq.}} \underline{\delta g_{\mu\nu}} + J^\mu_{\text{eq.}} \underline{\delta A_\mu}\right)$$

sources

$$W_{cons} = W_s + \int d^4x \sqrt{-g} \left(c_1 T^2 \Omega\cdot A + c_2 T\,(B\cdot A + \mu\Omega\cdot A) + \frac{C}{3}\mu\,\underline{(B\cdot A + \tfrac{1}{2}\mu\Omega\cdot A)}\right)$$

Chiral magnetic & chiral vortical effect in equilibrium

$$W_s = \int d^4x \sqrt{-g} \left(p(T,\mu,B^2) + \sum_{n=1}^{5} \underline{M_n}(T,\mu,B^2) s_n + O(\partial^2)\right)$$

5 thermodynamic transport coefficients

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

Variation of the …

… constitutive relations (1-point functions)

$$\boxed{G^R_{T^{\mu\nu},J^\alpha}} = \frac{\delta}{\delta A_\alpha} T^{\mu\nu}_{\text{on-shell}}[A,g]$$

$$\boxed{G^R_{J^\mu T^{\alpha\beta}}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\alpha\beta}}\left(\sqrt{-g}\, J^\mu_{\text{on-shell}}[A,g]\right)$$

➡ **Hydrodynamic 2-point functions**

[Ammon,Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

Recall Onsager relations

$$G^R_{TJ} = \eta_T \eta_J G^R_{JT}$$

… equilibrium generating functional

$$\delta W_s[A,g] = \int d^4x\sqrt{-g}\left(\tfrac{1}{2}\boxed{T^{\mu\nu}_{\text{eq.}}}\underline{\delta g_{\mu\nu}} + \boxed{J^\mu_{\text{eq}}}\underline{\delta A_\mu}\right)$$

sources

$$W_{cons} = W_s + \int d^4x\sqrt{-g}\left(c_1 T^2\Omega\cdot A + c_2 T\,(B\cdot A + \mu\Omega\cdot A) + \frac{C}{3}\mu\,\underline{\left(B\cdot A + \tfrac{1}{2}\mu\Omega\cdot A\right)}\right)$$

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5 thermodynamic transport coefficients

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1. Hydrodynamics - Constitutive relations

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

[Ammon, Kaminski et al.; JHEP (2017)]
[Hernandez, Kovtun; JHEP (2017)]

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$\begin{aligned}
\mathcal{E}_{\text{eq.}} =\, & -p + T p_{,T} + \mu p_{,\mu} + (T M_{5,T} + \mu M_{5,\mu} - 2M_5)\, B{\cdot}\Omega \\
& + (T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1)\, s_1 \\
& + (T M_{2,T} + \mu M_{2,\mu} - \boxed{M_2)\, s_2} \\
& + \frac{4B^2}{T^4}\left(M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
& + \left(T M_{4,T} + \mu M_{4,\mu} + \frac{4B^2}{T^4} M_{1,\mu} + M_{3,\mu}\right) s_4\,, \qquad (2.10a)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{eq.}} =\, & p - \tfrac{4}{3} p_{,B^2} B^2 - \tfrac{1}{3}(M_5 + 4M_{5,B^2} B^2) B{\cdot}\Omega - \tfrac{2}{3}\left(M_2 + 2B^2 M_{2,B^2}\right) s_2 \\
& + \frac{4B^2}{3T^4}\left(M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
& + \frac{4B^2}{3T^4}\left(M_{1,\mu} - T^4 M_{4,B^2}\right) s_4\,, \qquad (2.10b)
\end{aligned}$$

$$\begin{aligned}
\mathcal{Q}^\mu_{\text{eq.}} =\, & -M_5 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\sigma B_\rho + (2M_5 - T M_{5,T} - \mu M_{5,\mu}) \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma T/T \\
& - M_{5,B^2} \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma B^2 + (M_{5,\mu} - 2p_{,B^2}) \epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma \qquad (2.10c)
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}^{\mu\nu}_{\text{eq.}} =\, & 2p_{,B^2}\left(B^\mu B^\nu - \tfrac{1}{3}\Delta^{\mu\nu} B^2\right) + B^{\langle\mu} B^{\nu\rangle}\left(M_{5,B^2} B{\cdot}\Omega + M_{2,B^2} s_2 + (M_{4,B^2} - \tfrac{1}{T^4} M_{1,\mu}) s_4\right) \\
& + B^{\langle\mu} B^{\nu\rangle} \frac{1}{T^4}\left(T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} - M_1 + T^4 M_{3,B^2}\right) s_3 + M_5 B^{\langle\mu}\Omega^{\nu\rangle} \\
& + 2M_2 B^{\langle\mu}\epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho \partial_\sigma B_\alpha + (T M_{2,T} + \mu M_{2,\mu} - M_2)\, B^{\langle\mu}\epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma T/T \\
& + M_{2,B^2} B^{\langle\mu}\epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma B^2 - M_{2,\mu} B^{\langle\mu}\epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho E_\sigma B_\alpha\,, \qquad (2.10d)
\end{aligned}$$

1. Hydrodynamics - Constitutive relations

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

[Ammon, Kaminski et al.; JHEP (2017)]
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$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$\begin{aligned} \mathcal{E}_{\text{eq.}} = & -p + T p_{,T} + \mu p_{,\mu} + (T M_{5,T} + \mu M_{5,\mu} - 2M_5)\, B\cdot\Omega \\ & + (T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1)\, s_1 \\ & + (T M_{2,T} + \mu M_{2,\mu} - \boxed{M_2)\, s_2} \end{aligned}$$

➡ **Complicated because of broken symmetries:**

- **~~Chiral symmetry~~ — microscopic chiral anomaly**
- **~~Parity~~ — axial chemical potential**
- **~~Time reversal~~ + ~~Spatial rotation symmetry~~ — strong magnetic field** $B \sim \mathcal{O}(1)$

➡ **Many novel transport effects**

Two types :

1.) Thermodynamic transport coefficients

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{xz}T^{yz}}(\omega=0, k_z\hat{\mathbf{z}}) = -2\, B_0^2 \boxed{M_2}$$

novel: perpendicular magnetic vorticity susceptibility

+ *3 novel thermodynamic transport coefficients* M_1, M_3, M_4

+ *3 chiral conductivities (CME / CVE / CTE) in equilibrium,*
$\propto$ *chiral anomaly,*
measured negative magnetoresistance in 3D Weyl semimetals

+ ...

e.g. [Huang et al; PRX (2015)]

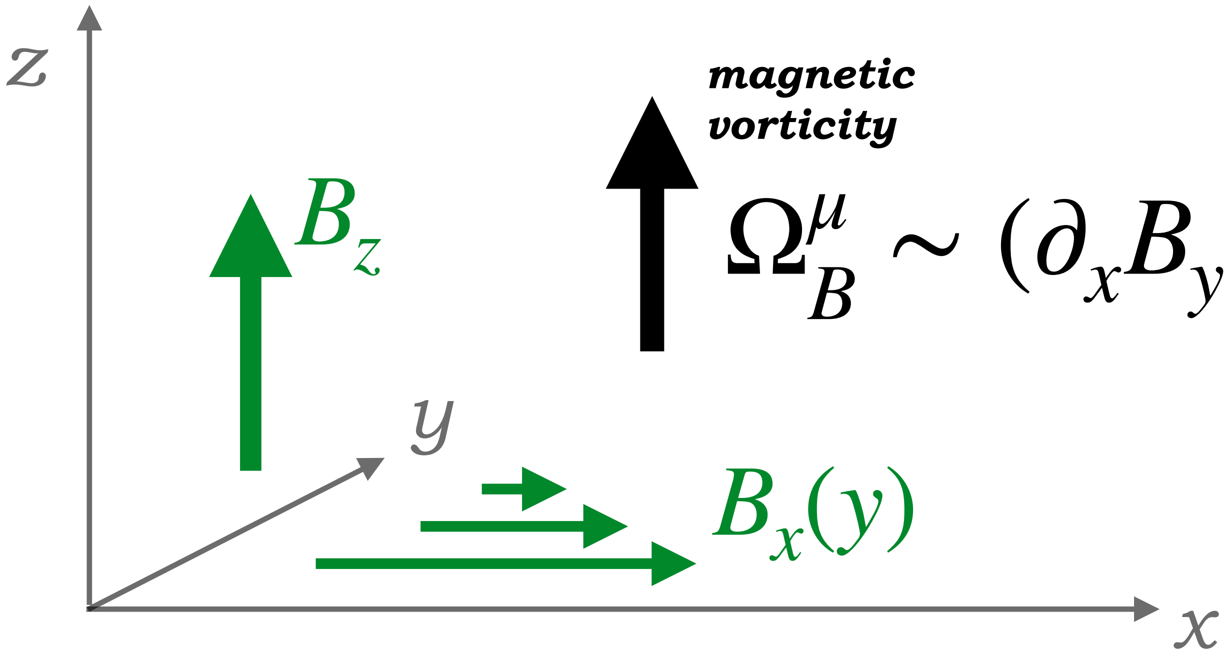
2.) Hydrodynamic transport coefficients

— boring example : $\frac{1}{\omega}\mathrm{Im}\, G_{J^z J^z}(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$

parallel charge conductivity

+ *3 novel hydrodynamic transport coefficients*

Perpendicular magnetic vorticity susceptibility M_2



$$\Omega_B^\mu \sim (\partial_x B_y - \partial_y B_x)$$

$$\boxed{\begin{aligned}\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} &\sim M_2 B\cdot\Omega_B \\ &\sim -M_2 B_z \partial_y B_x(y)\end{aligned}}$$

magnetic vorticity :

$$\Omega_B^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$$

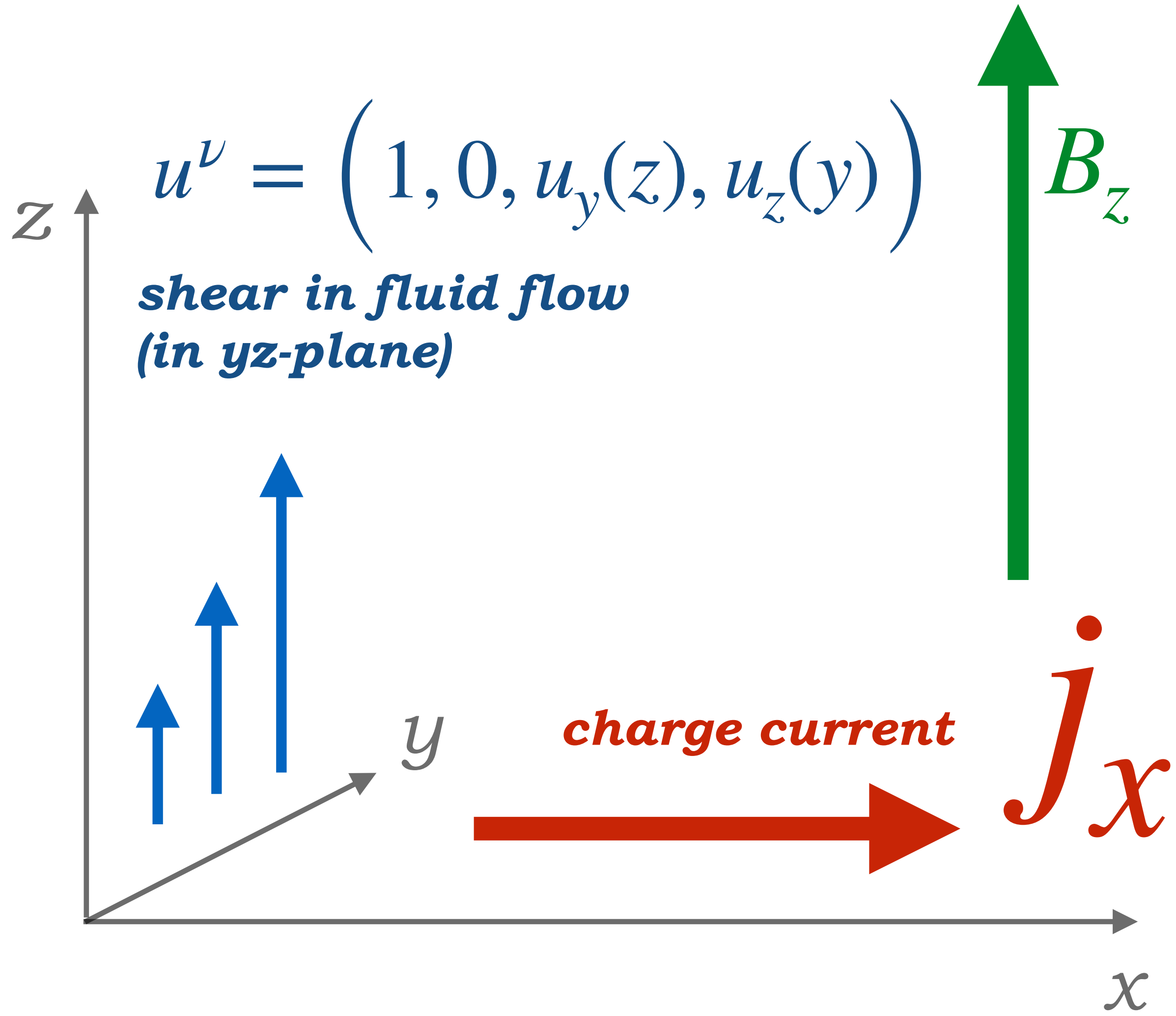
$$u^\nu = (1,0,0,0) + \mathcal{O}(\partial)$$

Recall tensor structure in W_s :

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

2. Hydrodynamics - c_{10} interpretation

Shear-induced Hall conductivity c_{10}



$$j_x \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

$$c_{10} \sim \frac{1}{\omega} \mathrm{Im}\, G_{T^{tx}\, T^{yz}}$$

- ➡ **novel Hall response**
- ➡ **non-dissipative**
- ➡ **interplay: shear-charge**

2. Hydrodynamics - 3 more novel coefficients

Shear-induced conductivity c_8

$$j_x \sim c_8(\partial_x u_z + \partial_z u_x)$$

Expansion-induced conductivities c_4 and c_5

$$j^\mu \sim \hat{b}^\mu(c_4 \nabla \cdot u + c_5 \hat{b}^\alpha \hat{b}^\beta \partial_\alpha u_\beta)$$

➡**Fluid flow gradients create charge currents**

3. Holographic model

➡ **use as holographic dual to charged state in strong *B***

➡ **values for transport coefficients in *N*=4 Super-Yang-Mills**

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A \wedge F \wedge F\right]$$

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes

[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
- asymptotically AdS_5
- (pseudo)spectral method (checked with shooting method)

3. Holographic model

Einstein-Maxwell-Chern-Simons equations of motion

$$R_{mn} = -4\, g_{mn} + \frac{1}{2}\left(F_{mo}\, F_n{}^o - \frac{1}{6}\, g_{mn}\, F_{op}F^{op}\right)$$

$$\nabla_m F^{mn} + \frac{\gamma}{8\sqrt{-g}}\, \varepsilon^{nmopq} F_{mo} F_{pq} = 0$$

Charged magnetic black brane ansatz

$$\begin{aligned} ds^2 \;&=\; \frac{1}{\varrho^2}\left[\left(-u(\varrho) + c(\varrho)^2\, w(\varrho)^2\right)\, dt^2 - 2\, dt\, d\varrho + 2\, c(\varrho)\, w(\varrho)^2\, dz\, dt \right. \\ &\left. \quad + v(\varrho)^2\, \left(dx^2 + dy^2\right) + w(\varrho)^2\, dz^2\right]\,, \\ F \;&=\; A_t'(\varrho)\, d\varrho \wedge dt + B\, dx \wedge dy + P'(\varrho)\, d\varrho \wedge dz\,, \end{aligned}$$

Near-horizon expansion

$$\begin{aligned} u(\varrho) &= (1-\varrho)\left[\bar{u}_1 + \mathcal{O}(1-\varrho)\right], & c(\varrho) &= (1-\varrho)\left[\bar{c}_1 + \mathcal{O}(1-\varrho)\right], \\ v(\varrho) &= \bar{v}_0 + \mathcal{O}(1-\varrho), & A_t(\varrho) &= (1-\varrho)\left[\bar{A}_{t\,0} + \mathcal{O}(1-\varrho)\right], \\ w(\varrho) &= \bar{w}_0 + \mathcal{O}(1-\varrho), & P(\varrho) &= \bar{P}_0 + \mathcal{O}(1-\varrho)\,, \end{aligned}$$

Temperature and entropy

$$T = \frac{|u'(1)|}{4\pi} \qquad\qquad s = 4\pi\, v(1)^2\, w(1) = 4\pi\, \bar{v}_0^2\, \bar{w}_0$$

Near-boundary expansion

$$u(\varrho) = 1 + \varrho^4 \left[u_4 + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho) \left[\frac{B^2}{6} + \mathcal{O}(\varrho^2)\right] ,$$

$$v(\varrho) = 1 + \varrho^4 \left[-\frac{w_4}{2} + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho) \left[-\frac{B^2}{24} + \mathcal{O}(\varrho^2)\right] ,$$

$$w(\varrho) = 1 + \varrho^4 \left[w_4 + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho) \left[\frac{B^2}{12} + \mathcal{O}(\varrho^2)\right] ,$$

$$c(\varrho) = \varrho^4 \left[c_4 + \mathcal{O}(\varrho^2)\right] + \varrho^8 \ln(\varrho) \left[-\frac{B^2}{12} c_4 + \mathcal{O}(\varrho^2)\right] ,$$

$$A_t(\varrho) = \mu - \frac{\rho}{2}\varrho^2 - \frac{\gamma B p_1}{8}\varrho^4 + \mathcal{O}(\varrho^6) ,$$

$$P(\varrho) = \varrho^2 \left(\frac{p_1}{2} + \frac{\gamma B \rho}{8}\varrho^2 + \mathcal{O}(\varrho^4)\right) ,$$

Energy-momentum tensor and charge current

$$\langle T_{\mu\nu}\rangle = \lim_{\varrho\to 0} \frac{1}{\varrho^2} \Big(-2K_{\mu\nu} + 2(K-3)\, g_{\mu\nu} + \ln(\varrho) \left(F_\mu{}^\alpha F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right) + \hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} + 8\ln(\varrho)\, h^{(4)}_{\mu\nu} \Big)$$

$$\langle J^\mu_{cons}\rangle = \lim_{\varrho\to 0} \left(\sqrt{-g} n_a g^{a\nu} F_{\nu\sigma} g^{\sigma\mu} + \frac{\gamma}{6}\epsilon^{\alpha\beta\gamma\mu} A_\alpha F_{\beta\gamma} + \ln\varrho \sqrt{-\hat{g}}\, \hat{\nabla}_\nu F^{\nu\mu} \right)$$

$$\langle J^\mu_{cov}\rangle = \lim_{\varrho\to 0} \left(\sqrt{-g} n_a g^{a\nu} F_{\nu\sigma} g^{\sigma\mu} + \ln\varrho \sqrt{-\hat{g}}\, \hat{\nabla}_\nu F^{\nu\mu} \right)$$

$$h^{(4)}_{\mu\nu} = \frac{1}{8}\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\rho\sigma} + \hat{R}\hat{\nabla}_\mu\hat{\nabla}_\nu\hat{R} - \frac{1}{16}\hat{\nabla}^2\hat{R}_{\mu\nu} - \frac{1}{24}\hat{R}\hat{R}_{\mu\nu} + \frac{1}{96}\left(\hat{\nabla}^2\hat{R} + \hat{R}^2 - 3\hat{R}_{\rho\sigma}\hat{R}^{\rho\sigma}\right)\hat{g}_{\mu\nu}$$

3. Holographic model

➡ **Charged equilibrium state**
➡ **external magnetic field**

$$\langle T^{\mu\nu}_{\text{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^{\mu}_{\text{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

$$u(\varrho) = 1 + \varrho^4\left[u_4 + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho)\left[\frac{B^2}{6} + \mathcal{O}(\varrho^2)\right],$$

$$v(\varrho) = 1 + \varrho^4\left[-\frac{w_4}{2} + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho)\left[-\frac{B^2}{24} + \mathcal{O}(\varrho^2)\right],$$

$$w(\varrho) = 1 + \varrho^4\left[w_4 + \mathcal{O}(\varrho^2)\right] + \varrho^4 \ln(\varrho)\left[\frac{B^2}{12} + \mathcal{O}(\varrho^2)\right],$$

$$c(\varrho) = \varrho^4\left[c_4 + \mathcal{O}(\varrho^2)\right] + \varrho^8 \ln(\varrho)\left[-\frac{B^2}{12}c_4 + \mathcal{O}(\varrho^2)\right],$$

$$A_t(\varrho) = \mu - \frac{\rho}{2}\varrho^2 - \frac{\gamma B p_1}{8}\varrho^4 + \mathcal{O}(\varrho^6),$$

$$P(\varrho) = \varrho^2\left(\frac{p_1}{2} + \frac{\gamma B \rho}{8}\varrho^2 + \mathcal{O}(\varrho^4)\right),$$

Energy-momentum tensor and charge current

$$\langle T_{\mu\nu}\rangle = \lim_{\varrho\to 0}\frac{1}{\varrho^2}\Big(-2K_{\mu\nu} + 2(K-3)\,g_{\mu\nu} + \ln(\varrho)\left(F_\mu{}^\alpha F_{\nu\alpha} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right)$$
$$+\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} + 8\ln(\varrho)\,h^{(4)}_{\mu\nu}\Big)$$

$$\langle J^\mu_{cons}\rangle = \lim_{\varrho\to 0}\left(\sqrt{-g}n_\alpha g^{\alpha\nu}F_{\nu\sigma}g^{\sigma\mu} + \frac{\gamma}{6}\epsilon^{\alpha\beta\gamma\mu}A_\alpha F_{\beta\gamma} + \ln\varrho\,\sqrt{-\hat{g}}\,\hat{\nabla}_\nu F^{\nu\mu}\right)$$

$$\langle J^\mu_{cov}\rangle = \lim_{\varrho\to 0}\left(\sqrt{-g}n_\alpha g^{\alpha\nu}F_{\nu\sigma}g^{\sigma\mu} + \ln\varrho\,\sqrt{-\hat{g}}\,\hat{\nabla}_\nu F^{\nu\mu}\right)$$

$$h^{(4)}_{\mu\nu} = \frac{1}{8}\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\rho\sigma} + \hat{R}\hat{\nabla}_\mu\hat{\nabla}_\nu\hat{R} - \frac{1}{16}\hat{\nabla}^2\hat{R}_{\mu\nu} - \frac{1}{24}\hat{R}\hat{R}_{\mu\nu} + \frac{1}{96}\left(\hat{\nabla}^2\hat{R} + \hat{R}^2 - 3\hat{R}_{\rho\sigma}\hat{R}^{\rho\sigma}\right)\hat{g}_{\mu\nu}$$

3. Holographic model

Green's functions for Kubo relations

$$\lim_{k_z \to 0} \frac{1}{k_z} \mathrm{Im}\, G_{\mathcal{O}_a \mathcal{O}^c}(\omega{=}0, k\,\mathbf{e}_z)$$

$$\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G_{\mathcal{O}_a \mathcal{O}^c}(\omega, \mathbf{k}{=}0)$$

Metric fluctuations (gauge field fluctuations not displayed)

$$\delta \langle \mathcal{O}_a \rangle (\omega, \boldsymbol{k}) = G_{\mathcal{O}_a \mathcal{O}^c}(\omega, \boldsymbol{k})\, \delta\phi_c(\omega, \mathbf{k})$$

Expand to linear order in frequency and momentum

$$h_{mn}(\varrho, k) = h^{(0)}_{mn}(\varrho) + k\, h^{(1)}_{mn}(\varrho) \qquad h_{mn}(\varrho, \omega) = h^{(0)}_{mn}(\varrho) + \omega\, h^{(1)}_{mn}(\varrho)$$

Near-boundary expansion

$$h^{(0)}_{mn}(\varrho) = h^{(0)}_{mn,\boldsymbol{s}} + \varrho^4\, h^{(0)}_{mn,\boldsymbol{v}} + B^2 h^{(0)}_{mn,\boldsymbol{s}} \varrho^4 \log(\varrho)$$
$$h^{(1)}_{mn}(\varrho) = h^{(1)}_{mn,\boldsymbol{s}} + \ldots + \varrho^4\, h^{(1)}_{mn,\boldsymbol{v}} + (B\, h^{(0)}_{mn,\boldsymbol{s}} + B^2\, h^{(1)}_{mn,\boldsymbol{s}})\, \varrho^4 \log(\varrho),$$

3. Holographic model

Green's functions for Kubo relations

$$\lim_{k_z\to 0} \frac{1}{k_z} \mathrm{Im}\, G_{\mathcal{O}_a \mathcal{O}^c}(\omega{=}0, k\,\mathbf{e}_z)$$

$$\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\, G_{\mathcal{O}_a \mathcal{O}^c}(\omega, \mathbf{k}{=}0)$$

Helicity	Fluctuation modes
2	$h_{12},\ h_{11}-h_{22}$
1	$h_{t1},\ h_{13},\ a_1,\ h_{z1}$
	$h_{t2},\ h_{23},\ a_2,\ h_{z2}$
0	$h_{tt},\ h_{t3},\ h_{33},\ h_{11}+h_{22},\ h_{zt},\ h_{z3},\ h_{zz},\ a_t,\ a_3,\ a_z$

Metric fluctuations (gauge field fluctuations not displayed)

$$\delta\left\langle \mathcal{O}_a \right\rangle(\omega, \boldsymbol{k}) = G_{\mathcal{O}_a \mathcal{O}^c}(\omega, \boldsymbol{k})\, \delta\phi_c(\omega, \mathbf{k})$$

Expand to linear order in frequency and momentum

$$h_{mn}(\varrho, k) = h^{(0)}_{mn}(\varrho) + k\, h^{(1)}_{mn}(\varrho) \qquad h_{mn}(\varrho, \omega) = h^{(0)}_{mn}(\varrho) + \omega\, h^{(1)}_{mn}(\varrho)$$

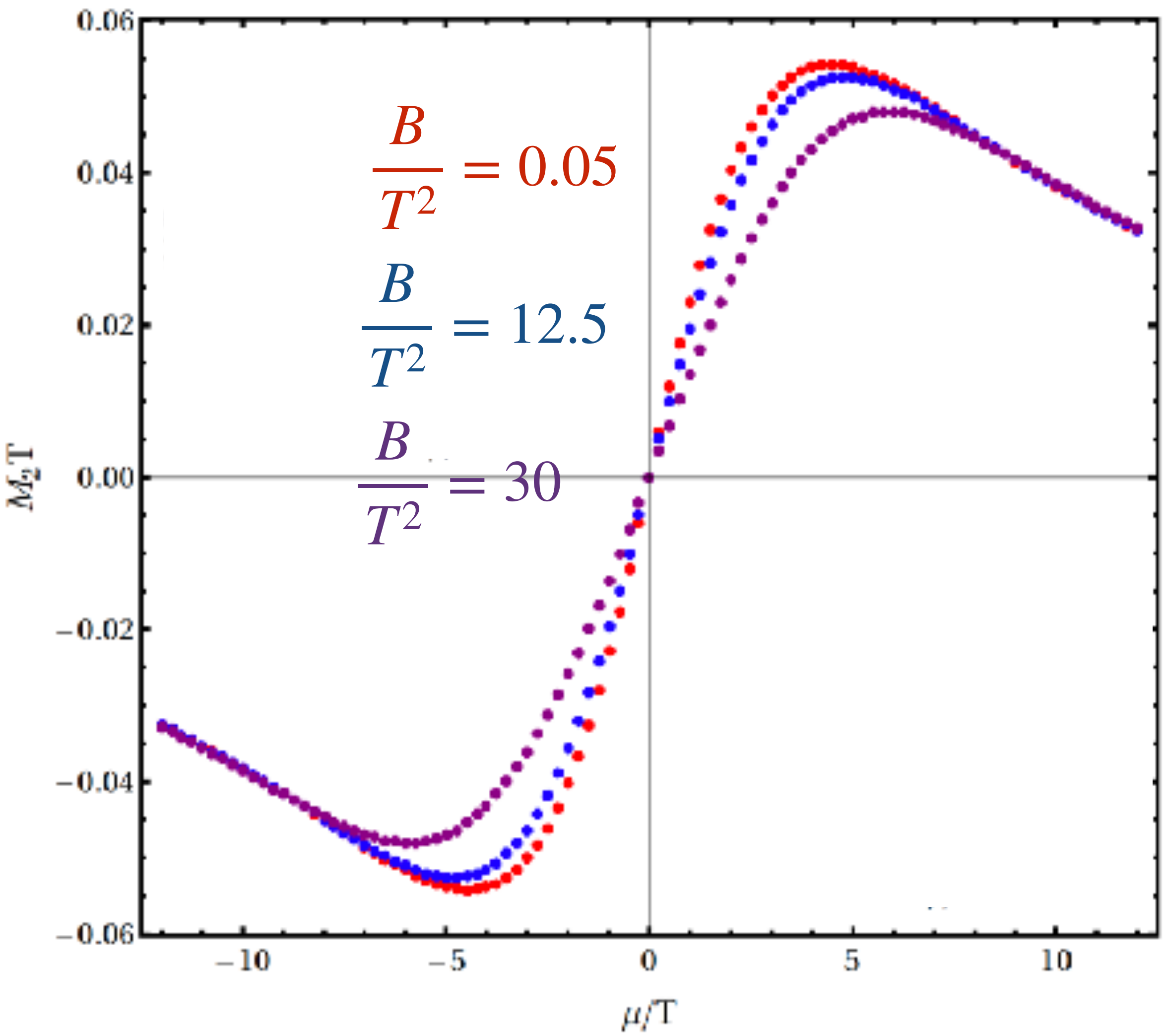
Near-boundary expansion

$$\begin{aligned} h^{(0)}_{mn}(\varrho) &= h^{(0)}_{mn,\boldsymbol{s}} + \varrho^4\, h^{(0)}_{mn,\boldsymbol{v}} + B^2 h^{(0)}_{mn,\boldsymbol{s}} \varrho^4 \log(\varrho) \\ h^{(1)}_{mn}(\varrho) &= h^{(1)}_{mn,\boldsymbol{s}} + \ldots + \varrho^4\, h^{(1)}_{mn,\boldsymbol{v}} + (B\, h^{(0)}_{mn,\boldsymbol{s}} + B^2\, h^{(1)}_{mn,\boldsymbol{s}})\, \varrho^4 \log(\varrho), \end{aligned}$$

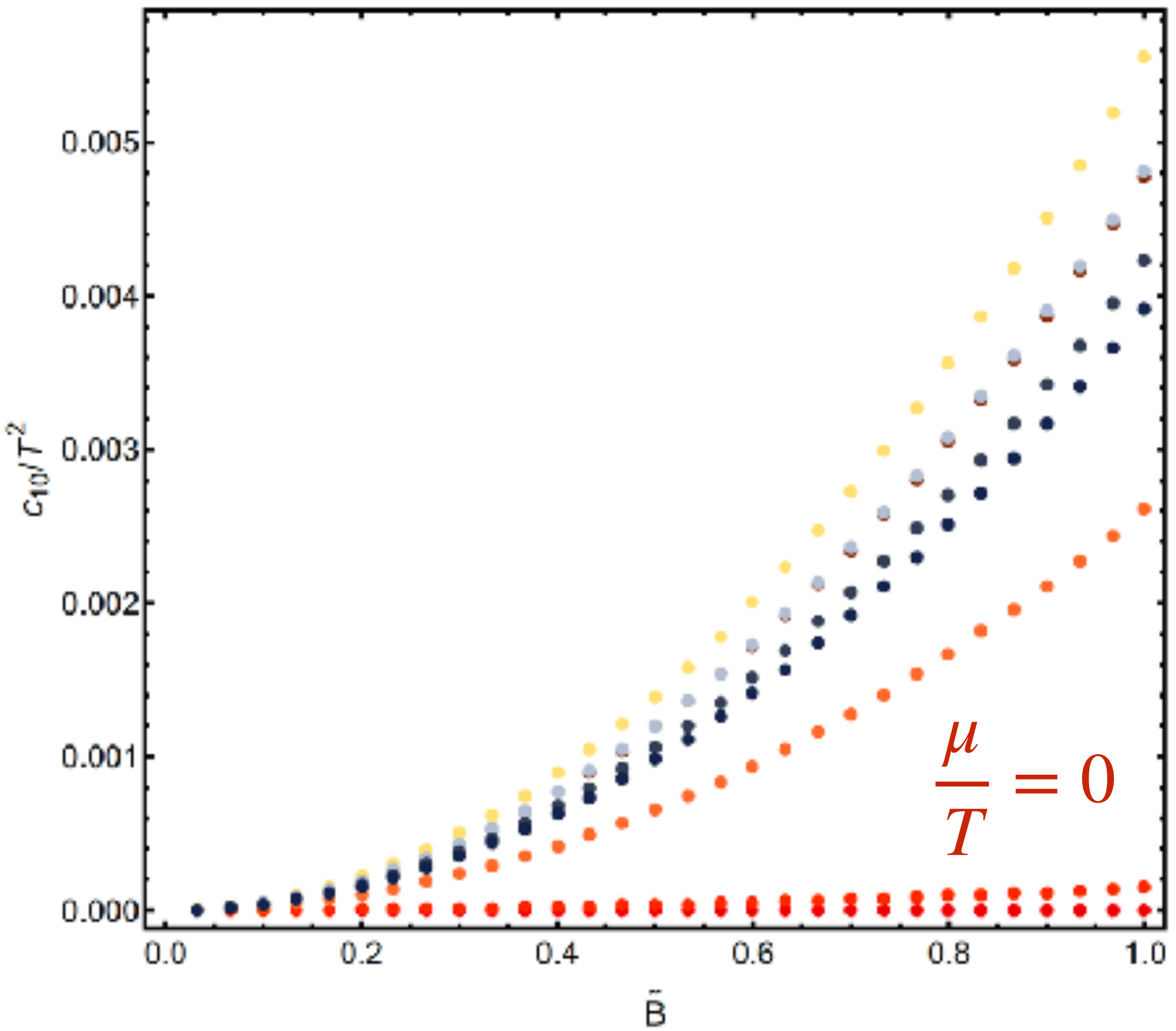
3. Holographic model - Results

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

Perpendicular magnetic vorticity susceptibility M_2



Shear-induced Hall conductivity c_{10}



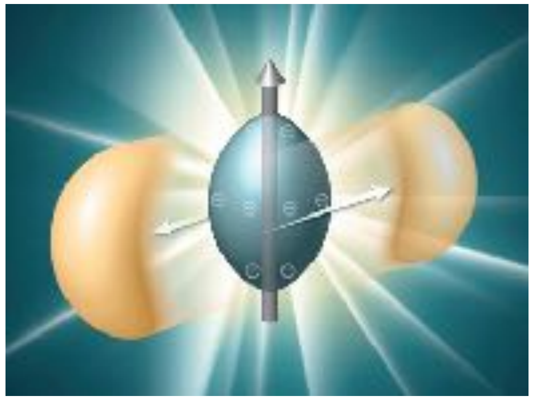
➡ **not zero, finite, Onsager satisfied**

➡ **Kubo formulae reasonable**

RECALL: 1. Hydrodynamics - Examples

Quark Gluon Plasma

- strong magnetic field B
- chiral anomaly
- chiral transport effects, e.g. ***chiral magnetic effect / wave***



[Fukushima,Kharzeev,Warringa; PRD (2008)]

[Kharzeev,McLerran,Warringa; Nucl.Phys.A (2008)]

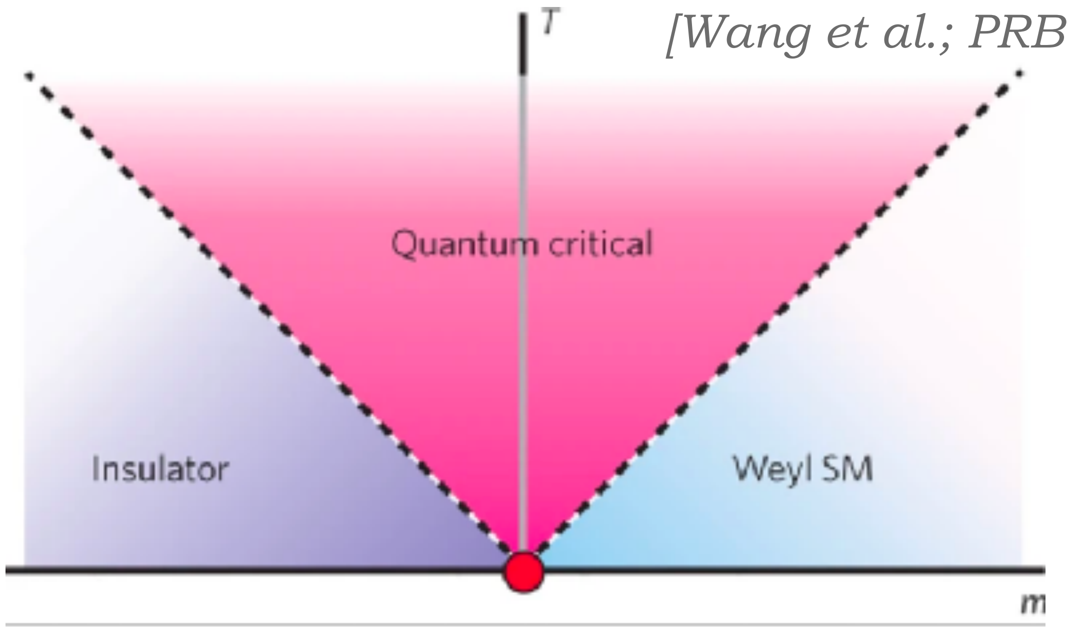
[Kharzeev,Yee; PRD (2011)]

Weyl-semimetals

- relativistic Weyl fermions + B
- chiral anomaly, ***CME / wave***
- quantum critical point in Weyl semimetals & in QGP?

[Yang et al.; Nature (2014)]

[Wang et al.; PRB (2019)]



All coefficients

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim_{\mathbf{k}\to 0}\lim_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{zz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	J_xT_{ty} (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	J^xJ^y (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	J^tT^{xx} (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	J^tT^{tt} (2.32)	+	+	-
M_4	magneto-electric susceptibility	J^tJ^t (2.32)	+	-	-

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
dissipative, hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_\parallel$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_\parallel$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	J^xJ^x (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	J^xJ^y (2.55e)	+	+	-
$\sigma_\parallel$	long. conductivity	J^zJ^z (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp \delta_{ab} + \tilde{\rho}_\perp \epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	J_xT_{xx} (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	J_zT_{zz} (2.57)	+	-	-
c_3		$c_5 = -3(c_3 + c_4)$	+	-	-

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Non-dissipative Hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx}-T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}$, $T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	J^xJ^x, J^xJ^y (2.54,2.53b,2.53c)	+	-	+

3. Discussion - Summary

Hydrodynamics

- (3+1)D hydrodynamics: charged chiral fluids in strong B
- 5 novel hydro transport coefficients (+3 thermo) at leading and sub-leading order in the hydrodynamic expansion
- as important as shear viscosity and charge conductivity
- Kubo formulae for 25 transport coefficients

Hydrodynamics

- far from equilibrium fluid dyn.

[Romatschke; PRL (2018)]
[Cartwright, Kaminski; JHEP (2019)]
[Wondrak, Kaminski, Bleicher; PRB (2020)]

- quantum chaos

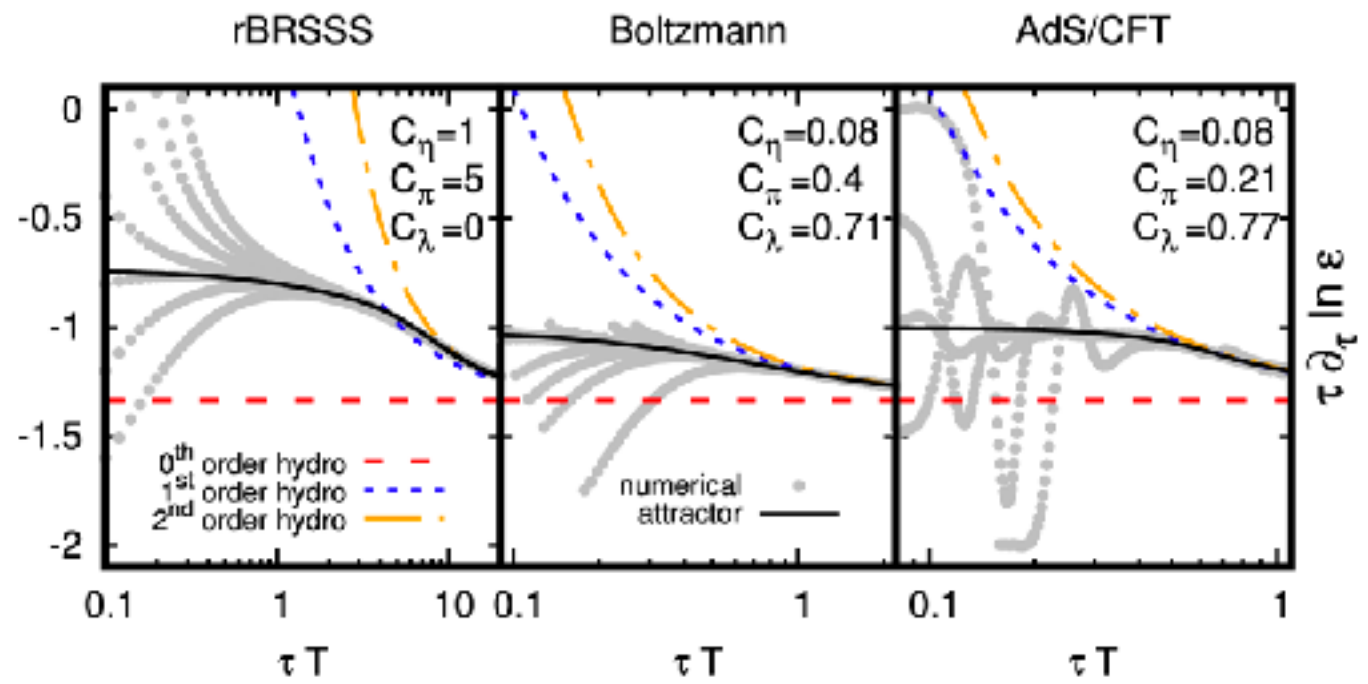
[Blake, Lee, Liu; JHEP (2018)]
[Grozdanov et al. (2019)]

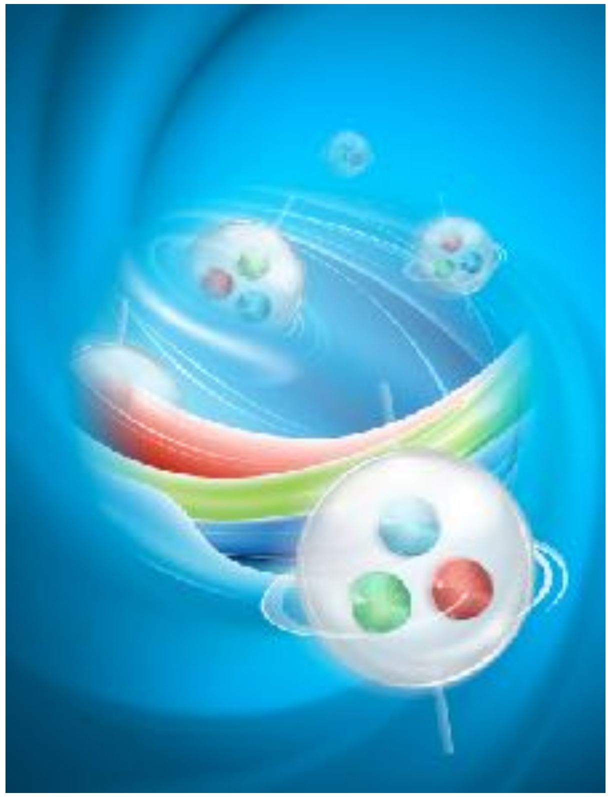
- convergence & stability

[Kovtun; JHEP (2019)]
[Grozdanov, Kovtun, Starinets, Tadic; PRL (2019)]
[Withers; JHEP (2018)]
[Heller, Janik, Witaszczyk; PRL (2013)]
[Heller, Spalinski; PRL (2018)]

- most vortical fluid

[Garbiso, Kaminski; JHEP (2020)]





[STAR; Nature (2017)]

Collaborators on these projects

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Thank you for listening!

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Casey Cartwright

APPENDIX

CPT symmetries

quantity	C	P	T
t	+	+	-
x^i	+	-	+
r	+	+	+
T, h_{tt}, T^{tt}	+	+	+
μ_A, A_t, J^t	+	-	+
μ_V, V_t, J_V^t	-	+	+
A_i, J^i	+	+	-
V_i, J_V^i	-	-	-
A_r	+	-	-
V_r	-	+	-
u^i, h_{ti}, T^{ti}	+	-	-
h_{ij}, T^{ij}	+	+	+
B^i	+	-	-
B_V^i	-	+	-
E^i	+	+	+
E_V^i	-	-	+
$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^\kappa$	+	-	-
$\int_i^f A \wedge F \wedge F$	+	+	+
$\int_i^f V \wedge F_V \wedge F_V$	-	-	+
u^t	+	+	+
generating functional W (axial $U(1)_A$)	+	+	+

More thermodynamic transport coefficients

Magneto-thermal susceptibility M_1 :

$$\mathcal{E}_{\rm eq} \sim M_1 B^\mu \partial_\mu \left(\frac{B^2}{T^4}\right)$$

Magneto-acceleration susceptibility M_3 :

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_{3,B^2}\, B{\cdot}a$$

Magneto-electric susceptibility M_4 :

$$\mathcal{E}_{\rm eq} \sim M_{4,T}\, B{\cdot}E\,, \qquad \mathcal{P}_{\rm eq} \sim M_{4,B^2}\, B{\cdot}E$$

Magneto-vortical susceptibility M_5 :

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_5 B{\cdot}\Omega$$

Strong B thermodynamics

$$\boxed{B \sim \mathcal{O}(1)}$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly : $\langle T^{\alpha\beta}\rangle = \epsilon\, u^\alpha u^\beta + p\Delta^{\alpha\beta} + \tau^{\alpha\beta}$

Energy momentum tensor:

equilibrium heat current

$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^\mu \rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ new contributions to thermodynamic equilibrium observables

previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

EFT calculation: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly

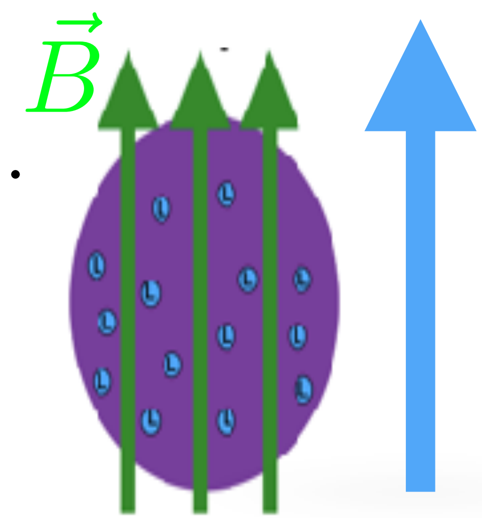
$$\partial_\mu J_A{}^\mu = C\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}\, F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Axial current with **weak** external B field:

$$\langle J_A{}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) + \xi_B B^\mu + \xi_V \Omega^\mu + \ldots$$

(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underbrace{u^\mu q^\nu + u^\nu q^\mu}_{\text{heat current}} + \tau^{\mu\nu}$$

measured in Weyl semi metals
e.g. [Huang et al; PRX (2015)]

neutron stars?
[Kaminski et al.; PLB (2014)]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!
[Landau, Lifshitz]
[Kadanoff; Martin]

Dispersion relations: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \underline{\frac{B n_0}{\epsilon_0 + P_0}} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \underline{\frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s}/\partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = \underline{v_0\, k} - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = \underline{v_+\, k - i\Gamma_+\, k^2} + \mathcal{O}(\partial^3)$$

$$\omega_- = \underline{v_-\, k - i\Gamma_-\, k^2} + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**
[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B\, T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$
$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$ *former sound modes*

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial\epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\,\frac{w_0\,\sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3C T_0 \mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1}{w_0}\,c_s^2\,\xi_V^{(0)},$$

$$v_0 = \frac{2BT_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument

[Son, Surowka; PRL (2009)]
[Neiman, Oz; JHEP (2010)]

Update: weak B hydrodynamics comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients

except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]

— take B=0 values of this model instead

weak B hydro prediction:

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\boxed{\eta}}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \boxed{\sigma}}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

Update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $\omega = \pm\frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}\boxed{(\sigma_\perp \pm i\tilde{\sigma})} - i\boxed{D_c}k^2$

parity-odd

weak B: $\omega = \mp\frac{Bn_0}{\epsilon_0+P_0} - ik^2\frac{\eta}{\epsilon_0+P_0} + k\frac{Bn_0\xi_3}{(\epsilon_0+P_0)^2} - \frac{iB^2\sigma}{\epsilon_0+P_0}$

Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $\omega = \pm k v_s - i\frac{\Gamma_{s,\|}}{2}k^2\,,$

$\omega = -iD_\| k^2\,,$

Anisotropic transport coefficients

$$D_\| = \frac{\boxed{\sigma_\|}w_0^2}{n_0^2\boxed{\chi_{11}} + w_0^2\boxed{\chi_{33}} - 2n_0w_0\boxed{\chi_{13}}}$$

parity-odd

weak B: $\omega_0 = v_0 k - iD_0 k^2 + \mathcal{O}(\partial^3)$

$\omega_+ = v_+ k - i\Gamma_+ k^2 + \mathcal{O}(\partial^3)$

$\omega_- = v_- k - i\Gamma_- k^2 + \mathcal{O}(\partial^3)$

Agreement in form

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2BT_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

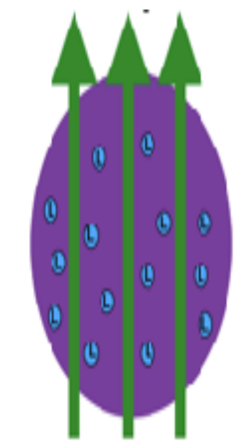
Action and background

Einstein-Maxwell-Chern-Simons action

chiral anomaly

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

$$S_{bdy} = \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} d^4x\,\sqrt{-\hat{g}}\left(K - \frac{3}{L} + \frac{L}{4}R(\hat{g}) + \frac{L}{8}\ln\left(\frac{\varrho}{L}\right)F_{\mu\nu}F^{\mu\nu}\right)$$



Magnetic black branes *[D'Hoker, Kraus; JHEP (2009)]*

- charged magnetic analog of RN black brane
- Asymptotically AdS5
- zero entropy density at vanishing temperature

$$ds^2 = \frac{1}{\varrho^2}\left[\left(-u(\varrho) + c(\varrho)^2\,w(\varrho)^2\right)\,dt^2 - 2\,dt\,d\varrho + 2\,c(\varrho)\,w(\varrho)^2\,dz\,dt + v(\varrho)^2\,\left(dx^2 + dy^2\right) + w(\varrho)^2\,dz^2\right]\,,$$

$$F = \underbrace{A_t'(\varrho)\,d\varrho\wedge dt}_{\text{charge}} + \underbrace{B\,dx\wedge dy}_{\text{magnetic field}} + P'(\varrho)\,d\varrho\wedge dz\,,$$

Correlators from infalling fluctuations

Problem: fluctuation equations are coupled (dual to operator mixing in QFT)

Numerical methods

- matrix method and shooting technique

[Kaminski, Landsteiner, Mas, Shock, Tarrio; JHEP (2010)]

$$G^{(ret)}(\mathbf{k}) = -2 \lim_{\epsilon\to 0} \mathcal{F}(\mathbf{k}, \epsilon)$$

$\Longrightarrow$ frequency and momentum

find independent solutions to coupled systems (pure gauge solutions)

- one-point functions technique and spectral methods

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; arXiv:2012.09183]

$$\langle \mathcal{O}_A \, \mathcal{O}_B \rangle \sim \frac{\delta \langle \mathcal{O}_B \rangle}{\delta \phi_A}$$

$\Longrightarrow$ analytic relations

find independent solutions to coupled systems (no pure gauge solutions)

Holographic result: equilibrium

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]

[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Holographic result: equilibrium

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

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- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & \frac{B^2}{4} + u_4 + 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ 4\,c_4 & 0 & 0 & 8\,w_4 + u_4 \end{pmatrix}$$

$$\langle J^{\mu}\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

$$\langle T^{\mu\nu}_{\mathrm{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB}B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB}B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^{\mu}_{\mathrm{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

➡ **agrees in form with strong *B* thermodynamics from EFT**